

MODELS FOR HOSPITAL CENSUS PREDICTION AND
ALLOCATION

By

KHANH-LUU THI NGUYEN

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MODELS FOR HOSPITAL CENSUS PREDICTION AND
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By

Khanh-Luu Thi Nguyen

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This dissertation presents models for hospital census prediction and allocation. Models to minimize the penalty costs for not having sufficient beds for individual services are used to describe the hospital bed allocation problem. An algorithm using a queueing theory approach for a system with a Poisson arrival process, an Erlang k service time distribution and a random number of servers is used to derive the bounds on the probability of the number of beds occupied in each service. The minimum expected cost allocation is evaluated by a heuristic algorithm.

The census prediction model is based on probability theory with the assumption that patients' lengths of stay are statistically independent. The mean residual length of stay function is used to evaluate the conditional probability of staying in the hospital given the number of days the patient has already spent in the hospital. A method using the correlation between unscheduled and scheduled admissions is applied to predict the number of daily unscheduled admissions for hospitals with a large percentage of scheduled admissions. A time series model is used to estimate the unscheduled admissions by

day of the week for hospitals with a significant number of unscheduled admissions. Models are tested at various hospitals with different operating settings. Effects of hospital characteristics on the census prediction are identified.

CHAPTER ONE

INTRODUCTION

The objective of this research is to develop and test models related to the allocation of hospital beds between services and models for the prediction and control of inpatient admissions. The purpose of these models is to develop bed allocations and census predictions which improve the operation of the hospital with respect to specified criteria. Among the general objectives of this research are the reduction of health care costs due to inefficient resource allocation and the improvement of patient care by fostering timely assignments of a patient to the hospital and service best suited for his needs.

Efficiency of hospital operation is directly dependent on the skill with which management utilizes available resources. One of the major operational controls on resource utilization is the control of inpatient admissions (49) because these admissions trigger the usage of virtually every resource in the hospital from cotton swabs to nuclear scanners.

The occupancy level of a hospital not only affects its financial viability but also influences the effectiveness with which it can deliver its services. Low occupancy results in high costs per patient day and eventually leads to a reduction in staff and other resources required for high quality patient care. Excess occupancy stresses available resources, overloads staff and facilities, and causes congestion and delays throughout the hospital. Also, as the occupancy reaches the maximum, patients

seeking admissions must be turned away. This results in further disruption of hospital operations as scheduled procedures are cancelled, causing inconvenience for the patient and the physician, and, in some cases, aggravating the patient's medical condition.

In addition to the problems caused by the level of occupancy (either too high or too low), high variance in occupancy creates its own problems. With high census variance, it is necessary to maintain staff and other resources at a level sufficient to handle peak demand. This means that 90-95% of the time, depending on the risk level the administration adopts, these resources are under-utilized. This leads to further inefficiencies in operation.

Thus, admissions must be controlled to achieve an appropriate census level and to minimize census variance. However, achieving this control is difficult due to the random nature of requests for admissions and the random length of stay of the patients. Further, while the hospital administration can influence the rate and number of admissions, it cannot influence the discharge process which lies wholly within the physician's domain. Thus, control can be exercised on input but not on process or output. Further, control can be affected only over the elective portion of incoming patients. Emergency patients must be admitted immediately and urgent patients must be admitted (usually) within 24 hours. Only elective patients, who do not require immediate hospitalization, can be scheduled sufficiently far into the future to be useful as control variables. The problem of determining occupancy level and reducing occupancy variance therefore becomes one of finding optimal policies for scheduling elective admissions.

It is further recognized that many hospitals do not operate as monolithic units but as confederations of individual 'services', each controlled by an area chief. Services such as pediatrics, OB/GYN, medicine, psychiatry, and others, often function as hospitals within hospitals. Each service has an allocation of beds and admits patients to its own beds. If these beds are filled the service must borrow beds from services with empty beds or turn patients away. The probability that a service has sufficient beds is a function of the allocation of beds to each service and the patient demand for the service. Because it affects the response of the total hospital, bed allocation is considered as a sub-problem in this research. Models are developed to assist in decisions concerning the allocation of beds to services. Once the allocation decision has been made, the hospital can be compartmentalized and models of the admissions process can be tailored to each service.

There are five chapters in this dissertation. Chapters One and Five contain the introduction and conclusions of the research. Chapter Two reviews the relevant literature concerning the allocation of hospital beds, control of admissions, and related questions such as length of stay estimation and the patient arrival process. Chapter Three focuses on the allocation of hospital beds among services. Models are constructed to find an allocation that can minimize hospital operating costs such as the costs of turning patients away, of having patients in a common pool, and of having patients in borrowed beds. In these models, the state probability distribution function of the number of patients in each service is derived. These state probabilities, together with the relative costs of turning a patient away, of having a patient in the common pool or in a borrowed bed, make up the objectives of the allocation model. A sample

hospital is used for evaluating the relative costs and determining the allocation.

Chapter Four concentrates on the control of elective admissions. A census prediction model is developed based on the principal components of the admissions process. The model uses probability theory to estimate the expected census on any day in the future based on the current census, the scheduled reservations, the emergency arrivals and the patient length of stay. Different methods for predicting emergency arrivals and daily discharges are described in Chapter Four. The census prediction models are tested at various hospitals. From the results of these tests, the characteristics of each hospital and their effects on census prediction are identified. An ideal operation setting for the census prediction model is defined based on these characteristics. A brief look at policies for scheduling elective patients is also presented with suggested questions for future research.

CHAPTER TWO

LITERATURE REVIEW

The problem of admissions control has received considerable attention in the Operations Research literature. This chapter briefly surveys the relevant literature dealing with bed allocation, admissions control, length of stay estimation and related questions. A summary of the models and their assumptions is presented in Table 2.1. Section 2.1 discusses the literature related to the problem of bed allocation. Section 2.2 reviews research concerning the admissions process, control and modeling. Sections 2.3 and 2.4 study the literature on the length of stay and the patient arrival process, respectively.

2.1 Bed Allocation

Blumberg (11) and Weckwerth (70) predicted bed needs for a distinctive patient facility using the assumption that the daily census was Poisson distributed. Blumberg developed a table, based on the Poisson distributed daily census, that could determine the number of beds assigned to services to result in a fully occupied facility on an average of one day in 10, one day in 100, or one day in 1,000. Weckwerth formulated the proportion of time that an exact number of beds is filled as follows:

$$P[b] = (ADC)^b \frac{e^{-ADC}}{b!} ,$$

where ADC is the average daily census. Thus, both Blumberg and Weckwerth have used the Poisson distribution for planning the size of a hospital

Table 2.1
Summary of Admissions Models in Literature

Author (Year)	Model	Theory	Decision Variables	Decision Policies	Arrival	Length of Stay	Assumption	Comments
Bartley (56)	Daily census	Queueing		Poisson	Neg. Exp.	Poisson distributed daily census	Restricted assumptions	
Bainbridge (60)	Census prediction	Probability		Neg. Binomial	Log normal	Poisson distributed daily census	One-day prediction	
Thompson (60)	Bed need	Empirical data				Poisson distributed daily census		
Björnberg (61)	Bed need	Empirical data	Probability of fully occupied facility	Number of beds needed for the facility	Poisson (Em)	Poisson distributed daily census	Restricted assumptions	
Heckwith (65)	Bed need				L-usage			
Young (65)	Rate control	Queueing	Interarrival time between electives	Waiting time between electives	Erlang (Elec.)	Neg. Exp.	Restricted assumptions and open-loop control	
	Adaptive control	Queueing	Census level	Number of electives called	Poisson (Em)	Neg. Exp.	Existing waiting queue, instantaneous arrival of electives	
Resh (57)	Scheduling surgical patients	Probability forecast	Census levels	Number of patients scheduled			Unrealistic assumptions on electives.	
							Batch scheduling	Non-stochastic method

Table 2.1 (continued)

Author (year)	Date!	Theory	Decision Variables	Decision Policies	Arrival	Length of Stay	Assumption	Comments
Eberle (68)	Adaptive Control	Queueing	Census level, 3, and fixed scheduled number per day of the week	Poisson (λ_m)	-Existing patient queue -Discharge conditional on census is Poisson distributed	Same as comments on young		
Thomas (68)			predicting Markovian recovery probability		-Markovian state transition probability			
Goldstein (68)	Scheduling Simulations	-fixed page schedule -estimated length of stay	No emergency admissions					
Goldman (69)	Scheduling Simulation	Probability of overflow	No overflow					
Goldsby (69)	Scheduling Discrete Variables	Number of patients	Geometric and "asca" distributions				Restricted assumption	
Koester (70)	Scheduling Variables	Number of beds occupied	Random				non-realistic work-over assumption	
		Number of scheduled patients	Geometric				Schedule over planning horizon	

Table 2.1 (continued)

Author (Year)	Model	Theory	Decision Variables	Decision Policies	Arrival	Length of Stay	Assumption	Comments
Connors (70)	Scheduling electives	Simulation predicted census		No emergency			Continuous scheduling system, and no emergency admissions	
Zaldivar (70)	Bed allocation	Markovian probability	Number of beds to each service	Poisson	neg. exp.		Restricted assumptions	
Fishare [1] (71)	Scheduling surgical patients	Forecast census					Batch scheduling	Same as comments on Resh
Jackson (71)	Bed allocation	Probability of turning patients away		Poisson	Neg. exp.		No interaction among services	Restricted assumption
Kao (72)	Predicting recovery progress	Queueing Markov					-Markovian transition probability -holding time at each state	Same as comments on Thomas
Briggs (72)	Scheduling patients	Probability	A desired census level vs. predicted census				physicians' estimates	
Shonick & Jackson (73)	Scheduling patients	Queueing	Census 'level'				Neg. exp.	Restricted assumptions
Kushner & Chen (72)	Scheduling patients	Simulation	Census 'levels' (no elective, 'owed per day of the week')	Poisson			By day of admission	Evaluation of policies based on number of admissions and average bed utilization

Table 2.1 (continued)

Author (Year)	Model	Theory	Decision Variables	Decision Policies	Arrival	Length of Stay	Assumption	Comments
Wancock (74)	Scheduling patients	Simulation	"allowances" on services, and weekend				-Normal approximation for census distribution	
Swain (74)	Scheduling patients	Probability	Predicted census	Number of scheduled patients			-Exact census distribution	
Rubenstein (75)	Scheduling patients	Probability	Predicted census	Number of scheduled patients			-Normal approximation for census distribution	
Barber (75)	Scheduling patients	Probability	Predicted census	Dynamics policies for number of scheduled patients			-Stochastic demand of beds	

facility to accommodate a given average daily census with a predetermined probability of overflow. The assumption of a Poisson distributed daily census requires that every admission to the hospital is a random occurrence that is independent of every other admission, the length of stay has a negative exponential distribution, and the bed capacity is infinite. These requirements have restricted the applicability of this methodology.

Thompson and Fetter (65), recognizing the difficulty of assuming a Poisson distributed daily census, introduced a computer simulation to predict bed requirements for a maternity ward, for any given patient load and any desired service level. Service level was defined as the proportion of patients for which no extraordinary action would have to be taken as they progressed through the various facilities. Thompson and Fetter used accumulated data as input to the model and compared the output statistics to the actual ones for a period of 30 days to validate the simulation model. The simulation output on the number of beds occupied for different patient input rates was used to determine the number of beds required for a service. Thompson and Fetter also studied the sensitivity of the model with respect to an increase in admission rate, to elective inductions and to a change in length of stay. They found that the service level decreased with an increasing admission rate. By introducing an elective induction policy that allows a certain number of patients to be admitted to the hospital at two specific times of the day, they found an increase in bed utilization when compared to utilization without an elective induction policy. They concluded the method of scheduling was an important determinant of average occupancy. The simulation also showed that the number of beds occupied decreased as length of stay decreased, but the variance in the number of beds occupied remained the same.

Another simulation for bed allocation was done by Goldman, Knappenberger, and Eller (27). Goldman et al. investigated the effects of various beds-to-service and beds-to-room policies using simulation. Goldman considered two kinds of services: unrestricted services which were permitted to use beds in other services, and restricted services which were only allowed to use their own beds. Goldman introduced policies to allocate sufficient beds to restricted services to meet demand 95%, 85% and 75% of the time; the remaining beds were allocated to unrestricted services by an average demand for beds. The average bed utilization was tested for significant difference between these policies. It was found that the bed utilization of restricted services decreased significantly as the number of beds increased. The unrestricted service showed no significant difference in bed utilization for the above policies. Goldman suggested the following criteria for evaluating these policies: total bed utilization, patients waiting for admission, transfer problems resulting from any type of patient segregation, emergency patients placed in temporary beds, and patient care quality for patients placed in another service.

Zaldivar (76) used a Markovian model to derive the probability distribution of the number of beds occupied for each system state, with a state representing a different bed allocation. In his model, Zaldivar assumed patient arrivals were Poisson distributed, and patient lengths of stay were negative exponentially distributed. Zaldivar attempted to define bed allocation policies to minimize the expected number of borrowed beds in the system. A heuristic algorithm was used to solve the minimizing problem. The assumptions on patient arrival and patient length of stay distributions allowed Zaldivar to achieve analytic results. However, the

assumption of negative exponentially distributed length of stay does not generally hold for patients of all hospital services.

Jackson (36) used the M/M/x/x queueing system (Poisson arrivals to x identical servers with a negative exponential service distribution and no waiting line) to develop bed allocation policies such that the penalty of turning patients away was minimized. Jackson constructed an optimizing problem which can be solved by dynamic programming methods as follows:

$$\text{Min } \sum_{i=1}^N w_i \lambda_i P_i(x_i)$$

$$\text{s.t. } \sum_{i=1}^N x_i = B,$$

where w_i is a weighting factor for turn-away cost of service i , λ_i is the patient arrival rate to service i , and $P_i(x_i)$ is the probability of all x_i beds of service i full. The probability function $P_i(x_i)$ is given by the well-known Erlang loss formula. In his research, Jackson assumed that each service operated independently from one another. This assumption has restricted the applicability of Jackson's model. Jackson's approach to the bed allocation problem will be used in this research for developing different models where the independence assumption can be relaxed.

Singh (61) developed a procedure to allocate beds to services such that the operating costs were minimized. Three types of costs were considered: a fixed cost associated with the initial set-up cost of building the hospital, a holding cost associated with maintaining empty beds, and a shortage cost associated with refusing admission to a patient. Singh presented the following model that minimized the expected cost of providing m_k beds to the k^{th} disease.

$$\text{Min } \sum_{k=1}^N A_k \delta_k + c_1^k \sum_{v_k=0}^{m_k} (m_k - v_k) p(v_k) + c_2^k \sum_{v_k=m_{k+1}}^{\phi_k} (v_k - m_k) p(v_k)$$

$$\text{s.t. } \sum_{k=1}^N m_k = B,$$

where A_k = the fixed cost for disease k ,

$$\delta_k = \begin{cases} 0 & m_k = 0 \\ 1 & m_k > 0 \end{cases},$$

c_1^k = the holding cost for disease k ,

c_2^k = the shortage cost for disease k ,

v_k = the random variable for the demand of beds of disease k ,
and

$p(v_k)$ = the probability function of the demand of beds.

Singh used the assumption that the probability function of the demand of beds followed a Poisson distribution. Singh also gave a detailed analysis to evaluate the shortage costs in his research. Singh studied all possible courses of action available to the patient who faced a shortage situation such as being assigned a bed in a different ward, being sent to another institution, and going home to wait for admission. For each case, Singh considered the costs involved including both the cost to the hospital and the cost of the patient. The assumption of Poisson distributed demand for beds has restricted the applicability of this model as in the models of Blumberg and Weckwerth presented earlier. Other papers related to bed allocation are (4,10,18,31,32,45,53,68).

2.2 Census Process

Three fundamental factors of hospital census processes which have appeared in most of the literature are hospital bed occupancy levels, patient lengths of stay, and patient arrivals.

Early research on the number of patients in a hospital was done by Bailey (3), Thompson and Fetter (66), and Blumberg (11). In these studies, daily census levels were assumed to follow the Poisson distribution. Bailey assumed that the number of admissions was Poisson distributed and the patient length of stay was negative exponentially distributed. Under these assumptions, Bailey used the statistical theory of queues to show that the daily census was Poisson distributed. Thompson and Fetter compared the theoretical occupancy as obtained from a Poisson distribution, and the actual number of patients in a delivery suite over a period of 30 days. Thompson found that there was sufficient agreement between the two series of values and concluded that the number of patients in the delivery suite facilities was Poisson distributed. Blumberg based the assumption on his previously unpublished studies of data from several hospitals which indicated that daily census levels on a distinctive patient facility were generally Poisson distributed. Blumberg also showed that the daily census of over-crowded facilities with long waiting lists are not Poisson distributed. Therefore, the assumption of Poisson distributed daily census only applies to a certain distinctive patient facility.

Balintfy (5) predicted census analytically. Using a probabilistic approach, he predicted a hospital census one day in advance. Balintfy assumed that patient admissions followed a negative binomial distribution, and patient length of stay a lognormal distribution. The next day's census was derived from the current census and the convolution between the admission and discharge distributions. An approximation of the census distribution was made by assuming a normal distribution with the estimated census mean and variance. Balintfy's approach for predicting census is most useful when theoretical distributions can be shown to adequately approximate the admission and length of stay distributions. Frequently,

this is not the case. Also, a longer horizon for predicting census levels would be more useful for hospital planning.

Young (74,75) proposed a way to control census levels in a typical hospital ward. He developed queueing models, where hospital beds were considered as identical servers, patients as customers, and patient length of stay as service time. The hospital care unit then resembled a queueing system with finite number of servers, and two independent arrivals streams--one elective and one emergency. The emergency arrival process was assumed to be Poisson and the patient length of stay negative exponentially distributed. Young introduced different assumptions for the elective arrival process in two models: the rate control model and the adaptive control model. In the rate control model, the elective arrival process was assumed to follow an L-stage Erlang distribution with no waiting line. Steady state probability distributions of the number of patients in the hospital were found to be independent of L , the Erlang parameter. In the adaptive control model, elective arrivals were controlled by means of a control bed occupancy level B . When the number of occupied beds dropped below level B , elective arrivals were scheduled to bring the census level up to B . When the census levels were above B , only emergency patients were admitted. In the adaptive control model, Young assumed that patients formed a sufficiently long waiting queue and would be available immediately to enter the hospital upon scheduling. Steady-state probability distributions of the number of patients in the hospital were developed for the adaptive control model. The assumptions for the patient arrival process, the patient length of stay distributions, and the instantaneous availability of scheduled patients have limited applicability of the models. Moreover, the census control of the rate control model uses only the mean interarrival

time between two admissions and does not take advantage of any information about the state of the hospital system.

Resh (56) developed a scheduling approach for surgical patients using a probabilistic model to estimate the census mean and variance over a planning horizon period. Resh treated the scheduling problem in two stages. The first stage was concerned with the optimal type and number of patients to admit during the scheduling horizon, given a forecast of bed and surgical suite capacities, and considering prescheduled and nonscheduled admissions. This stage was described by a linear programming problem which minimized the expected waiting time per patient subject to the requirements that all admission requests had to be scheduled, total admissions of prescheduled and scheduled patients should not exceed the bed capacity, and the total surgical operation time should not exceed the number of available hours of the surgery suite with a probability of at least P. The second stage was concerned with daily optimal allocation of inpatients to operating rooms, given the admission data from the first stage decisions. This allocating program attempted to minimize overtime and idle time in the operating room. Both stages of the scheduling problem were planned to be executed daily. Resh's model has the advantage that no assumption is made on the types of distribution for patient arrivals and patient length of stay. However, Resh's batch method of scheduling requires an existing queue of elective patients and a deterministic number of patients scheduled for each day. Resh's model does not take into account the effect of additional scheduled patients on bed occupancy levels of the succeeding days.

Eberle (21) extended the work of Young in the adaptive control model to a more general formulation. Eberle was able to consider day-to-day effects on admissions and discharges. Eberle assumed that the number of elective

patients was constant for a given day of the week unless the number of unoccupied beds was less than the number of patients scheduled for admission. She also assumed that the distribution of the number of discharges per day, conditional on the census of the previous night, was Poisson distributed with mean values taken from historical averages for discharge data. Eberle then estimated the census-after-discharge distribution and derived the probability that some emergency patients would be turned away. These two quantities were suggested for measuring the effects of a policy change on the number of elective admissions admitted daily. Eberle's model as Young's model, requires an existing patient queue. Furthermore, the assumption that the discharge distribution, conditional on the census, is Poisson has restricted the generality of the model.

Thomas (64) developed a Markovian model for predicting the recovery progress of a particular class of patients--the coronary patients. States of the system corresponded to the states of the patients' health. Since the probability of transition to one recovery state depends on how long patients have been in the former state and is not memoriless as in Markovian systems, Thomas established three recovery phases within each of the recovery states. The probability of transition to one recovery phase has Markovian characteristics. With these phases, the number of patients in each state were approximated from the Markovian model. Thomas's model uses a Markovian assumption which is not realistic since a patient's future movements depend on his past history and not just on his current state of health. Moreover, the states of the system which are the states of the patients' health are not well-defined and will cause difficulty in implementing the model.

Robinson et al. (58) developed a simulation dealing sequentially with requests for admission, scheduling, and cost evaluation of three basic scheduling systems. Each elective patient entered the scheduling system with information on a desired admission date, flexibility of the possible admission day, potential length of stay in the hospital, and hospital service demand. The first scheduling system was termed the "filled page" method in which patients were scheduled to enter the hospital until a page had been filled or a set number of patients had been scheduled. The second scheduling system was based on the estimated patient length of stay to carry the expected census in the hospital out to some fixed horizon. The third method was called the "PT" (probability table) version where information about the conditional probability of the actual length of stay was used. The simulation evaluated the performance of these three different scheduling rules using the relative costs of empty beds, of hospital overflow and of turning prospective patients away. Robinson concluded that the estimated length of stay was the best technique. In the simulation, Robinson explicitly excluded emergency admissions which eliminated the random nature of the system. Thus, if emergency admissions were included, Robinson would probably find the PT method more useful.

Bithell (8) presented a class of discrete time models: nonhomogeneous Markov chains. The model is a discrete analog of Young's adaptive control model. The census was represented by a discrete time Markov chain, where the census levels were the states of the chain. The length of stay was distributed according to the geometric distribution. Bithell also generalized the length-of-stay distribution from a geometric to a Pascal distribution and expanded the scheduling horizon from a one-day to a several-day period. Bithell's model has two weaknesses: the restricted

assumption for the length of stay distribution and the Markovian assumption for the census movements.

Kolesar (42) formulated a Markovian decision model for treating the problem of scheduling elective admissions. Kolesar's model is a discrete time generalization of Young's adaptive control model and is almost identical to Bithell's model. Kolesar also used a Markov chain to represent census movements with random inputs and a geometric length of stay distribution. The only difference in Kolesar's model is that he gave the optimal control decisions for admissions scheduling, which were derived from the Markovian model, by controlling x_t , the number of beds occupied at time t. The problem was formulated as a linear programming problem with the objective function in terms of x_t . The Markovian decision model has predictive characteristics for a finite planning horizon by introducing more variables for states and decision rules for the problem. Kolesar's model has the same problems as Bithell's model: the restricted assumption for length of stay distributions and the Markovian census movements and unreasonable size.

Connors (16) used an admission scheduling algorithm in a real-time simulation model to schedule patients into a hospital. He used a probabilistic approach almost identical to the one developed by Resh to calculate the census mean and variance, and approximated the census distribution by a normal distribution. In the model, Connors excluded emergency admissions by assuming that a certain number of beds had been reserved for emergency use. The objective function of the model attempted to balance the patient's preference for day of admission and the type of facility with the hospital's desire to achieve an allocation providing the most revenues generated by the patient's admission.

Finarelli (23) developed a probabilistic model, based on that of Resh, for surgical patients. Finarelli included additional constraints on room and service capacity. Finarelli used hospital-oriented objectives instead of Resh's patient-oriented objectives since he argued that elective patients were those whose admission can be delayed at no medical cost. Finarelli introduced a heuristic algorithm for scheduling elective surgical patients to optimize the utilization of surgical beds and operating suite facilities. In summary, Finarelli's model is a modification of Resh's model, and still has Resh's weakness in the non-stochastics aspect of the model.

Kao (37,38,39,40,41) introduced a semi-Markovian model to describe patient movements through various hospital care zones. Kao's model modifies Thomas' model to handle system states to obtain Markovian characteristics. In Kao's semi-Markovian model, whenever a patient enters a state i , he will stay T_{ij} days in state i before making a transition to state j with the probability p_{ij} . Using this model, Kao predicted the number of patients in each hospital care zone which could be used for planning nursing staff and hospital facilities. Kao's model has the same weakness as Thomas' model in the definition of the states of health which limits the usefulness of this model. Moreover, the semi-Markovian model requires even more information than the Markovian model in order to construct holding time distributions.

Briggs (12) concentrated on inpatient scheduling to stabilize occupancy, thus reducing the probability of under and over-staffing in the service. A heuristic scheduling algorithm to minimize the discrepancy between actual and desired census for a given level of delays in admission was presented. The forecast of census depended on the expected number of non-emergent and

of emergent admissions for each day of the week, the number of scheduled reservations, the midnight census, and the estimate length of stay of patients. Using a recursive relation to estimate census after discharge for successive days, the model sequentially assigned a number of admissions each day to bring the expected census to some desired level after allowing for emergencies. Briggs' use of physicians' estimates of length of stay for predicting the number of patients discharged each day requires a high degree of cooperation from the medical staff. Moreover, Briggs' method of estimating the census does not take into account the fact that the probability of a patient remaining in a hospital for a certain number of days depends on the number of days that he has already spent in the hospital.

Shonick and Jackson (60) developed a queueing model combining Young's two models. The emergency arrival process and elective arrival process were both assumed to be Poisson distributed. The length of stay was assumed to be distributed negative exponentially for all patients. When the number of occupied beds reached a given level B , all elective arrivals were queued and only emergency arrivals were admitted. Shonick and Jackson derived the expected occupancy and the expected number of lost patients for steady-state solutions. Shonick and Jackson's assumption on the distribution of patient length of stay limits the usefulness of the model.

Kushner and Chen (44) built simulation models for studying different scheduling policies. The length of stay data were collected for emergencies and electives by the day of the week of admission. The policy used in admission was to admit scheduled patients until a bed capacity N was attained and urgent patients until B bed capacity was obtained ($B > N$). Emergency patients were always admitted. The simulation was run with different policies for the number of reservations accepted on each day of

the week. Kushner and Chen evaluated these policies by the number of admissions and average bed utilization.

Hancock et al. (30) tested different scheduling policies for an overbedded hospital. The system consisted of a set of policy rules, called "allowances." Each service had an "allowance" (or limit) on the number of scheduled patients. Weekend allowances were also considered in the system. Simulation was used to evaluate how a particular set of values for allowances met the objectives of the hospital.

Swain (62) has proposed a more elaborate predictive model for census. Statistical analyses were done on the empirical length of stay distributions for each service. The predicted census was derived from the current number of patients together with their length of time already in the hospital, the scheduled reservations for elective patients, and the expected number of emergency admissions for each day of the week. Since Swain dealt with a large population, he approximated the census distribution by a normal distribution with the estimated mean and variance derived from three above elements. Swain used the objective of maximizing average occupancy subject to bed overflow constraints based on the normal census approximation to derive policies on the number of scheduled elective patients for each day.

Rubenstein (59) introduced a model almost identical to that Swain's. The analyses on length of stay data were done for admitting diagnoses, age, and sex of patients. Rubenstein also found the exact census distribution by using the convolution of probability distributions of the three elements cited in Swain's model. Rubenstein concluded that the exact and normally approximated results did not differ significantly, even though the computational efforts for an exact result were 10 times more than that for an approximate one.

Barber (7) developed a model for census estimation and elective patient scheduling. He considered the case of dynamic decisions and that the requests for admission are processed on a continuous basis. The model consists of a two-step algorithm: 1) estimation of future census, and 2) optimal scheduling of elective patients. The census estimation is based on a mathematic development of the equations similar to the one developed by Swain (62). The census estimator consists of all census components: current patients, previous elective patient reservations, emergency admissions, transfers, and additional reservations. Barber introduced a method where stochastic admission requests may be processed. He considered the actual additional admissions, s_i , instead of the allowed additional admissions, e_i , i.e.,

$$s_i = e_i \quad \text{for requests greater than availability, and}$$

$$s_i < e_i \quad \text{for requests less than availability.}$$

Barber assumed that the probability distribution $f[s_t | e_t]$ was known and proceeded from this assumption to derive the optimal scheduling for maximizing census levels. Barber separated the optimal scheduling into two steps: a short-term sub-optimization for the decision period (1, T), and a modification on these decisions using a discount for future to account for the sequential nature of the decision process. Barber has considered the problem of scheduling patients a step closer to the real world situation. However, the probability function $f[s_t | e_t]$ on the actual admissions conditioned on the allowed admissions is difficult, if not impossible, to obtain.

Additional papers dealing with census prediction and admissions control include (20,22,24,33,50,52,69,71,73).

The models for census prediction used by Swain, Rubenstein, and Barber are based on probabilistic assumptions and provide a general approach to the census prediction problem. The use of empirical data on length of stay has made the models applicable to any hospital. However, the empirical data can also introduce too much noise which may effect the accuracy of the prediction. The approach of Swain, Rubenstein, and Barber will be used in this research to predict census along with a method to smooth out noise in empirical data on length of stay.

2.3 The Length of Stay Estimation

Early studies (2) on the length of stay indicated that the average length of stay was longer for males than females, for low-income groups than high-income groups. The average length of stay was also longest among persons aged 75 years and older. Another study (64) showed that the average length of stay for persons discharged by death was nearly double that for those discharged alive. In both discharge status groups, the average length of stay was greatest for persons 65 years of age and older.

Robinson et al. (57) studied various methods for predicting patient lengths of stay with less uncertainty, including use of diagnostic information, physicians' estimates, and nurses' discharge predictions. Physicians' estimates prior to admission, revised by additional estimates after admission, were found to be a useful method for reducing uncertainty in the length of stay of patients. The difficulty with this method of prediction is that physicians' estimates of length of stay are not usually available.

McCorkle (47) studied potential factors that would affect the length of stay. McCorkle compared the length of stay of patients who had a physician directly responsible for their hospital care (private patients)

with those who did not (staff patients). McCorkle found that the differences in the duration of hospitalization between private and staff patients varied considerably among hospital department and did not appear to be explained by variations in age, race, sex, or method of paying for care (Blue Cross vs. non-Blue Cross patients). McCorkle (48) examined the reason for the prolonged preoperative stay of a group of patients who underwent initial surgery two or more days following admission. He found that neither age nor classification as a private or staff patient was as important as admission status in predicting whether or not surgery would occur without delay. In one-third of the cases studied, delays were due to the fact that patients had been admitted to a nonsurgical department.

Gustafson (29) demonstrated five methodologies for predicting hospital length of stay. Three of these methodologies gave a point estimate of length of stay based on physicians' subjective opinions, while two gave a probability distribution for length-of-stay based on empirical data. In a statistical comparison of these five prediction methods, Gustafson concluded that the Bayesian model, which combined the impacts of data complexes such as day of admission, diagnostic information, and age and sex of patients on the hypothesized length-of-stay, was the most accurate technique, especially when attending physician's estimates on length of stay were not available.

Bithell and Delvin (9) studied the lengths of stay of a group of surgical patients to ascertain the extent to which discharges could be predicted. The estimates on the residual lengths of stay of individual patients were made by responsible clinicians two or three times each week and were recorded together with a degree of certainty attached to each estimate. Bithell found that the initial length of stay estimates together

with continuous predictions of discharges reduced variability in the discharge predictions. The problem with this method is that the estimates of length of stay are not always available especially with "hard cases" where uncertainty in length of stay is acute.

A report (15) on the length of stay of patients in short-term hospitals indicated that the average stay of patients varied considerably with age. The average length of stay of patients under one year of age was significantly greater than that of patients greater than one-year-old. Thirteen and fourteen-year-olds had a longer average stay than patients in the surrounding age group. From age eighteen to age twenty-six the average stay increased steadily with increasing age. This study was done with a sample taken from patients with all diseases in PAS (Professional Activity Study) hospitals during the first six months of 1970.

Another study (34) was done on short-term hospitals registered by the American Hospital Association of the seasonal effects on the length of stay of patients. Fluctuations caused by seasonality were found, tending to obscure long-term trends and cyclical movements. The study suggested that seasonally adjusted data, obtained through seasonal indexes, would be helpful for short-range and intermediate-range planning. Each monthly index characterizes the level of activity during that month as a percentage of the annual average.

Altman et al. (2) investigated the effects of different variables on length of stay such as mental status factors, diagnoses, sex, race, and marital status. They found that diagnostic categories were the best predictor of length of stay. The mental status of patients (depression or anxiety), marital status (married or widowed), and sex were also good predictors for length of stay.

Posner and Lin (54) studied the effect of patient age on length of stay. Their findings suggested that the variation in length of stay was not well accounted for by differences in the ages of individual patients. Patients of the same age had lengths of stay which varied much more than patients of different ages, even when diagnostic variables were taken into account.

2.4 Arrival Process

Other studies have concentrated on patient arrivals. Balintfly (5), as discussed previously, developed a negative binomial distribution to model inpatient admissions. Young (75), Thompson and Fetter (65), and Weckwerth (70) assumed a constant Poisson arrival process for emergency admissions. Swartzman (63), in a statistical analysis of patient arrivals in a Michigan hospital, concluded that the arrival processes were Poisson with arrival rates that differed significantly from segment to segment of the day but not from day to day over weekdays. The weekend arrival process was dropped from consideration since the greater part of patient arrivals occurred on weekdays.

Chen (13) and Fries (26) provide additional information concerning aspects of the admissions system.

CHAPTER THREE

THE BED ALLOCATION PROBLEM

3.1 Introduction

A hospital consists of many professional services; each deals with a medical specialty such as internal medicine, psychiatry, ophthalmology, and orthopedics. A practice that many hospitals use is to group hospital beds into sections or wards, each assigned to a service. The grouping of patients into a service section has the advantage of placing common types of patients together. Moreover, the physicians can concentrate on working within a section which can save their time and travel. Each section can also be equipped for a specialty which affords better quality of care for the patients. Due to the stochastic nature of the demand for beds, however, the grouping of patients in services often causes a situation where some services under-utilize their allotment of beds, while others have more patients than the assigned number of beds. The underbedded services with the demand for beds generally higher than their bed availability can resolve this problem in three different ways: 1) refuse admission to overflow patients; 2) borrow beds from under-utilized services for their overflow patients; or 3) use beds in a central bed pool, if such a pool exists, for their overflow patients. A central pool is a common ward with a fixed number of beds which can be used to accommodate the patients of any underbedded service. Solutions 2 and 3 cause a dilution in the grouping of common type patients. The control of elective admissions can be used to keep the occurrence of this situation to a minimum. However, with an unbalanced

allocation, the control of admissions might delay patient admissions. If a deferred patient chose to go to another hospital, hospital revenue would be reduced. Therefore, the allocation of hospital beds to services should be examined before any control is imposed on elective admissions. There three solutions to the problem of underbedded services are discussed in detail in following paragraphs.

"Bed borrowing" creates administrative problems such as additional record keeping, additional patient transfers, a lack of grouping common type patients, and inconvenience for physicians. "Bed borrowing" also introduces an extra operating cost. This cost is due to the fact that many services equip their rooms for special care. Having a patient in a borrowed bed necessitates moving specific equipment while equipment already in the room becomes idle. Moreover, patient grouping into professional services allows hospital staff to develop specialized skills in the performance of patient care functions. Having a patient in a borrowed bed demands extra skills from the nursing staff, while their specialized skills are not used.

An alternative to "bed borrowing" is to have a central pool in the hospital. The existence of a central pool is justified when the cost of having patients in central pool beds is lower than the cost of having patients in borrowed beds for some services. The central pool beds are usually equipped at a minimum level so that the operating cost due to idle equipment is negligible.

When an overflow service cannot find a bed to accommodate its patients, the patients will be turned away. Turned away patients can be either sent home to wait for admission on another date or admitted to another hospital. Thus, turning patients away creates an immediate loss to the hospital and a long range loss when physicians start admitting their patients to other hospitals.

It may be desirable for the hospital to allocate beds to services to minimize the costs of turning patients away and of having patients in borrowed or central pool beds. Another objective of bed allocation can be to minimize the probabilities of turning patients away and of having patients in borrowed or central pool beds. In this research, descriptive models of bed allocation systems will be used to determine the optimal bed allocation policies using the minimum cost objective. The optimal bed allocation policies with minimum probabilities of turning patients away and of having patients in borrowed or central pool beds can be constructed similarly. Four different hospital systems will be studied: 1) a system with no interaction among its services; 2) a system with interaction among its services; 3) a system with a central pool and no interaction among its services; 4) a system with a central pool and interaction among its services.

For the four hospital systems presented here, methodology will be developed for the allocation of B beds among N professional services. Each professional service has its own arrival and length-of-stay distributions, and a fixed number of beds assigned to it. The costs associated with having patients in central pool beds, in borrowed beds, and with turning patients away will be used as weighting factors in the models. The turnaway cost for service j , W_j , represents the service's subjective judgment about the acceptable number of patients turned away. The cost, V_{ij} , of having a patient of service j in a service i bed and the cost, U_j , of having a patient of service j in a central pool bed represent a relative cost due to the inconvenience of displacing the patient from service j .

It is very difficult to assign any monetary value to these costs. It would be best to have these costs in terms of general parameters which can

be changed by the administrators. One service can be chosen as the basis for assigning values to these parameters. Values of the parameters for other services can be considered as multiples of the ones of the chosen service. The opinions of the administrators and medical staff will be used to determine these parameters, especially when the three costs are weighted against each other. A method for evaluating these relative costs is presented and applied to a sample hospital.

Section 3.2 describes the four different models for bed allocation. The state probability distribution function for these systems is derived in Section 3.3. Solution to the bed allocation problem is presented in Section 3.4 and the evaluation of relative costs in Section 3.5.

3.2 Development of Descriptive Models

Consider an N -service hospital with a number of beds, B . Each service j is assigned s_j beds, $\sum_{j=1}^N s_j = B$. The patient arrival process to each service j is Poisson distributed with rate λ_j . In the bed allocation study, the arrival process is assumed not to be under any kind of admissions control, that is, all requests for admission are satisfied, or even if it is controlled, the number of admissions to each service follows a Poisson distribution as shown in the study of Swartzman (63). In addition to the beds assigned to individual services, a hospital system with a central bed pool also has a number of beds s_{N+1} assigned to the central pool, i.e.,

$$\sum_{j=1}^{N+1} s_j = B, s_j \geq 0 \forall j.$$

Let C_j be the random variable for the number of beds occupied by patients of service j . $P_j(m)$, for $m=0,1,2,\dots,B$ is the probability distribution function of the random variable C_j .

3.2.1 Model 1. A model of a hospital system with no interaction among services.

Jackson (36) developed this model based on the assumption that all services operate independently. In other words, all overflow patients of a service are turned away. This simplified model does not accurately represent the real world where services can borrow beds. However, this model can be used to gain insight into more realistic models which will be discussed later.

The objective of this model is to minimize the expected cost of turning patients away where the turnaway cost, as discussed previously, is measured by a weighting factor w_j .

Let $P_j(s_j)$ be the probability that a patient arriving for service j will find all s_j beds of service j filled, i.e., the probability that an arriving patient for service j will be turned away:

$$P_j(s_j) = \Pr[C_j \geq s_j] .$$

The flow of patients in this system is summarized in the flow chart in Figure 3.1.

The expected rate at which patients are turned away from service j , given the mean arrival rate of patients to service j is λ_j patients/day, is

$$\lambda_j P_j(s_j) .$$

The expected penalty cost of turning patients away from service j is

$$w_j \lambda_j P_j(s_j) .$$

The total expected penalty cost of turning patients away from the hospital is

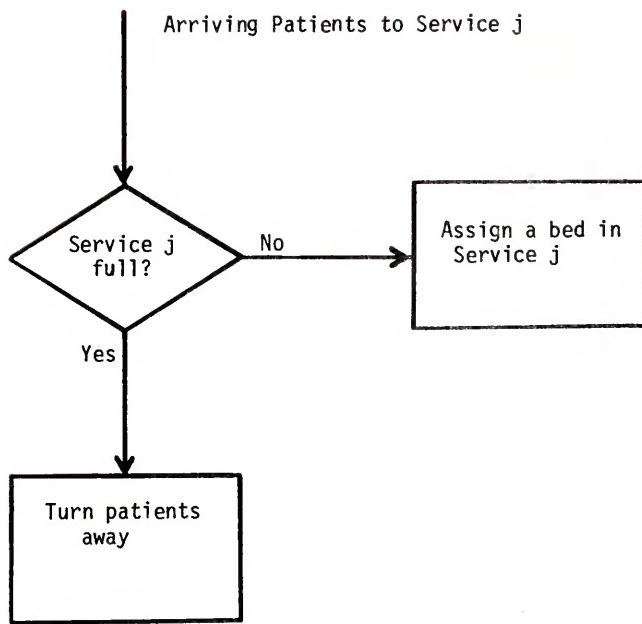


Figure 3.1 The flow of patients in Model 1.

$$\sum_{j=1}^N w_j \lambda_j p_j(s_j) .$$

The bed allocation problem for this model can be written as

$$\text{Min } \sum_{j=1}^N w_j \lambda_j p_j(s_j)$$

$$\text{s.t. } \sum_{j=1}^N s_j = B$$

$$s_j \text{ integer, } j=1, 2, \dots, N.$$

If the $p_j(s_j)$ functions are known, the problem is easily recognized as a simple dynamic programming problem of the knapsack variety.

3.2.2 Model 2. A model of a hospital system with interactions among services.

The assumption for this model is that the overflow patients of a service can be absorbed by any other service, if there is an available bed. If there is no bed available in the hospital, the overflow patient is turned away. In this model, it is assumed that a service can borrow beds from any other service, and the penalty cost of having its patient in any borrowed bed is independent of the service borrowed from, i.e.,

$$v_{ij} = v_j \quad \forall i.$$

In this system, the overflow patient is turned away only when the hospital is completely full. Therefore, the turnaway rate is independent of the allocation of beds among service. The objective of this system is assumed to be to minimize the expected cost of having patients in borrowed beds.

The flow of patients in service j is described in the flow chart in Figure 3.2.

If the probability distribution function, $P_j(m)$, of the number of beds occupied by service j is known, the expected rate of borrowed beds can easily be found:

$$\lambda_j P_j(s_j) \Pr[C_1 + \dots + C_j + \dots + C_N < B] .$$

The bed allocation problem for this model can be written as

$$\text{Min } \sum_{j=1}^N v_j \lambda_j P_j(s_j) \Pr[C_1 + \dots + C_j + \dots + C_N < B]$$

$$\text{s.t. } \sum_{j=1}^N s_j = B$$

$$s_j \text{ integer, } j = 1, 2, \dots, N .$$

The probability distribution function $P_j(m)$ will be derived in Section 3.3, together with solution techniques for the above optimization problem.

3.2.3 Model 3. A model of a hospital system with a central pool and without interaction among services.

In this system, each of the N professional services of the hospital is assigned a fixed number of beds. There is also a central pool (considered to be the $(N+1)^{\text{th}}$ service) which can provide a fixed number of beds to overflow patients from any service. Overflow patients of a service can be absorbed by the central pool if there are beds available, otherwise, they will be turned away. The objective is assumed to be to allocate beds to services so that the costs of having patients in central pool beds and of turning patients away from the hospital are minimized. Overflow patients

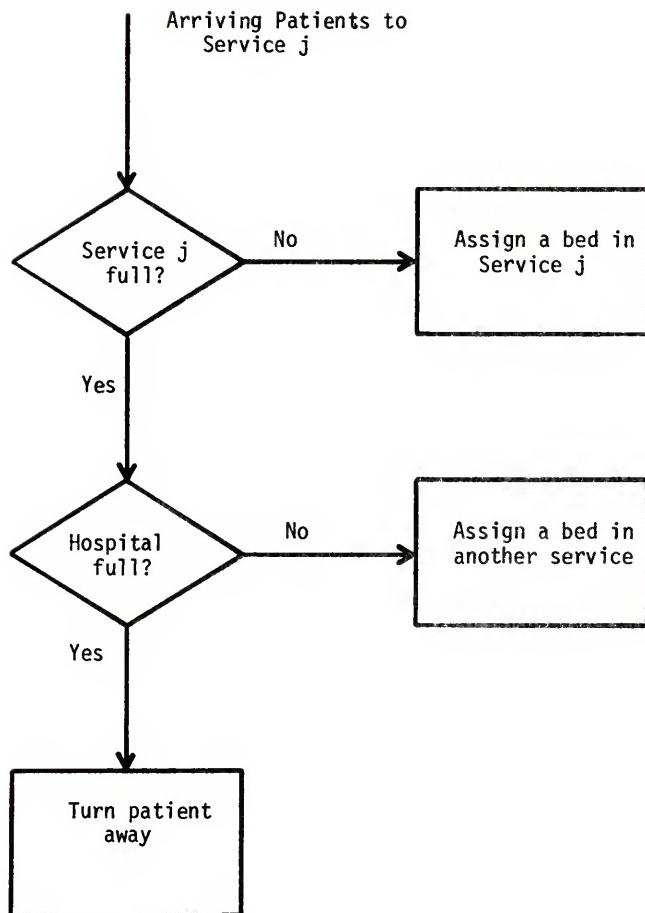


Figure 3.2 The flow of patients in Model 2.

of service j become the arriving patients of service j to the central pool. These patients will be turned away from the central pool if the central pool is full.

The general flow of patients of service j is summarized in the flow chart in Figure 3.3. The probability that an overflow patient of service j will find the central pool full can be written as

$$\Pr[C_{N+1} = s_{N+1} | C_j \geq s_j] = \sum_{m=0}^{s_{N+1}} \Pr[C_{N+1} = s_{N+1} | C_j = s_j + m] \Pr[C_j = s_j + m],$$

where C_{N+1} is the number of occupied beds in the central pool. The probability function of C_{N+1} is the convolution of overflow probabilities of all services. The right hand side of the above equation can be easily found if the probability distribution functions, $P_j(m)$, for all services are known.

The expected rate of overflow patients from service j that are turned away is

$$\lambda_j P_j(s_j) \Pr[C_{N+1} = s_{N+1} | C_j \geq s_j].$$

The expected rate of patients from service j to central pool beds is

$$\lambda_j \sum_{m=0}^{s_{N+1}} P_j(s_j + m) [1 - \Pr[C_{N+1} = s_{N+1} | C_j = s_j + m]]$$

The expected costs of having patients in central pool beds and of turning patients away are

$$\sum_{j=1}^N \{ U_j \lambda_j \sum_{m=0}^{s_{N+1}} P_j(s_j + m) [1 - \Pr[C_{N+1} = s_{N+1} | C_j = s_j + m]]$$

$$+ W_j \lambda_j P_j(s_j) \Pr[C_{N+1} = s_{N+1} | C_j \geq s_j] \} .$$

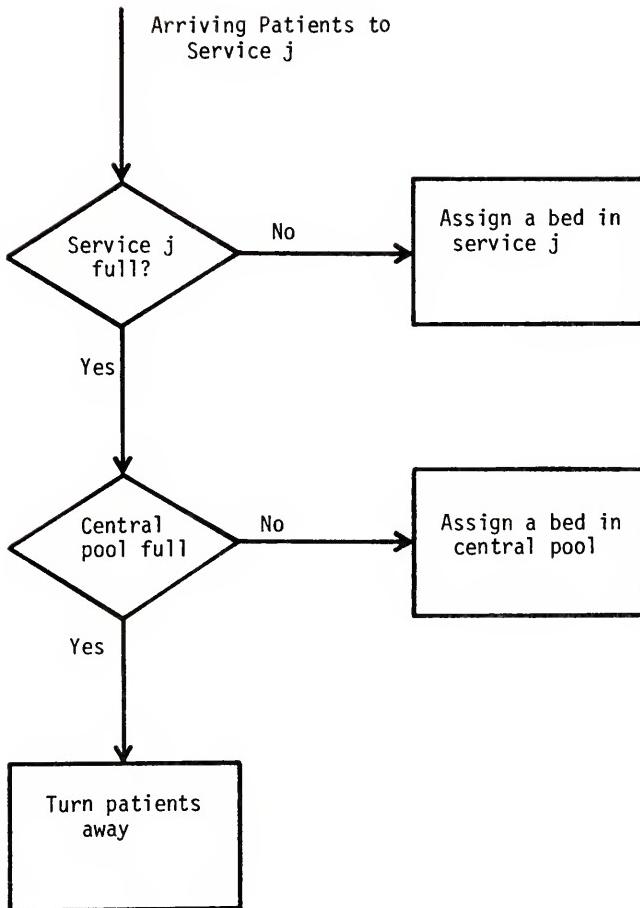


Figure 3.3 The flow of patients in Model 3.

The bed allocation problem for this model can be written as

$$\begin{aligned} \text{Min } & \sum_{j=1}^N \{ \lambda_j \sum_{m=0}^{s_{N+1}} P_j(s_j+m) [1 - \Pr[C_{N+1} = s_{N+1} | C_j = s_j+m]] \\ & + W_j \lambda_j P_j(s_j) \Pr[C_{N+1} = s_{N+1} | C_j \geq s_j] \} \\ \text{s.t. } & \sum_{j=1}^{N+1} s_j = B \quad s_j \text{ integer, } j=1, 2, \dots, N+1 . \end{aligned}$$

The probability distribution function $P_j(m)$ will be derived in Section 3.3. Solution techniques for the above optimization problem will also be presented.

3.2.4 Model 4. A model of a hospital system with a central pool and interaction among its services.

Model 4 is an extension of Model 3. When all central pool beds are filled, an overflow patient can be assigned to a bed borrowed from some other service. Patients are turned away from the hospital only when all beds in the hospital are filled. Therefore, the rate of turned-away patients does not depend on the allocation of beds to services. For this hospital system, the objective is assumed to be to allocate beds to services so as to minimize the total costs of having patients in central pool beds and in borrowed beds.

The general patient flow of service j is summarized in the flow chart in Figure 3.4.

The expected rate of overflow patients from service j that can be accommodated in central pool beds is

$$\lambda_j \sum_{m=0}^{s_{N+1}} P_j(s_j+m) [1 - \Pr[C_{N+1} = s_{N+1} | C_j = s_j+m]]$$

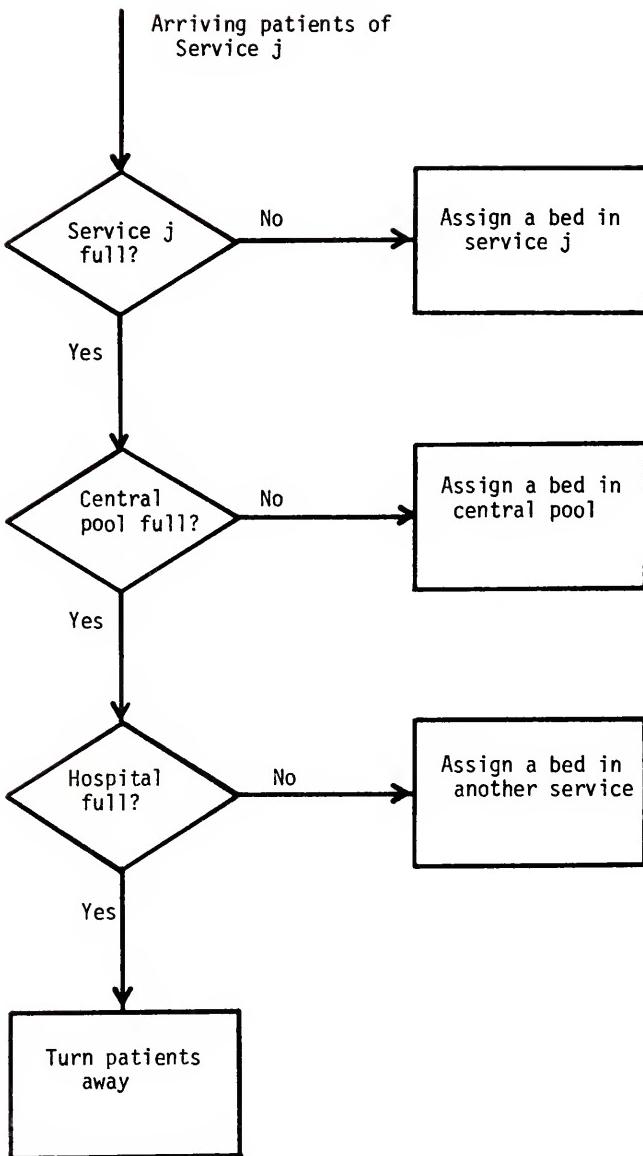


Figure 3.4 The flow of patients in Model 4.

The expected rate of overflow patients from service j that find all central pool beds filled but are able to obtain a borrowed bed is

$$\lambda_j P_j(s_j) \Pr[C_{N+1} \geq s_{N+1} | C_j \geq s_j] \Pr[C_1 + \dots + C_j + \dots + C_N < B].$$

The bed allocation problem for this model can be written as

$$\begin{aligned} \text{Min } & \sum_{j=1}^N \{ U_j \lambda_j P_j(s_j) [1 - \Pr[C_{N+1} \geq s_{N+1} | C_j \geq s_j]] \\ & + V_j \lambda_j P_j(s_j) \Pr[C_{N+1} \geq s_{N+1} | C_j \geq s_j] \Pr[\sum_{i=1}^N C_i < B] \end{aligned}$$

$$\text{s.t. } \sum_{j=1}^{N+1} s_j = B$$

$$s_j \text{ integer, } j = 1, 2, \dots, N+1.$$

If the probability function $P_j(m)$ is known, the objective of the above optimization problem can be evaluated and the solution found. The derivation of this probability function will be presented in Section 3.3.

3.3 Development of Probability Distribution Functions

Before proceeding further, notational definitions for the queueing systems are given for later use in the analysis.

$M/E_k/s_j/s_i$: The queueing system with Poisson arrivals to $s_j + s_i$, identical servers with Erlang k service time distribution and no waiting line. $P_j(m), m=0, 1, 2, \dots, s_j + s_i$ are the state probabilities for the system of service j .

$M/E_k/s_j/s_i^*$: The queueing system with Poisson arrivals to s_j continuously available servers plus s_i servers available on a random basis, each of the $s_j + s_i$ servers possessing the same Erlang k service time distribution,

and no waiting line. $P_j^*(m), m=0,1,\dots,s_j+s_i$ are the state probabilities for the system of service j .

Model 1 is easily seen to be equivalent to well-known queueing systems with Poisson arrivals to identical Erlang k servers and no waiting line. The state probabilities of service i in this system are the same as the ones of the $M/E_k/s_i/0$ queueing system.

In the following section, the state probability distribution for the systems in Models 2, 3, and 4 will be examined. The determination of state probabilities for these systems appears to be extremely difficult at best. In this research, no attempt is made to solve for state probabilities explicitly. Instead, portions of the system are related to queueing systems for which solutions are well known. In so doing, it is possible to determine upper and lower bounds for certain elements of the state probabilities. The bounds may then be successively tightened by a procedure which takes advantage of the relationship existing between services.

3.3.1 The $M/E_k/s_j/s_i$ Queueing System

The $M/E_k/s_j/s_i$ queueing system (Poisson arrivals to (s_j+s_i) identical servers with an Erlang k service distribution and no waiting line) is the basic vehicle of comparative analysis. The solution for the $M/E_k/s_j/s_i$ system is well-known (43,17,28,55):

$$P_j(m) = P_j(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!}, \text{ where}$$

$P_j(m) =$ the probability of m customers in the system, $m=0,1,\dots,s_j+s_i$,

$$P_j(0) = \frac{1}{\sum_{n=0}^{s_i+s_j} \left(\frac{\lambda_j}{\mu_j}\right)^n \frac{1}{n!}} = \text{the probability of no customer in the system,}$$

λ_j = the arrival rate of customers to the system, and

$1/\mu_j$ = the mean service time of customers in the system.

3.3.2 The Single Service $M/E_k/s_j/s_{N+1}^*$ Queueing System

The hospital system, if considered as a whole, is extremely complex. However, individual services of the hospital have identical structures. Overflow patients of a service can be absorbed by a central pool, if there are beds available, otherwise, the patients are turned away. Therefore, the single service system can be considered as an $M/E_k/s_j/s_{N+1}^*$ queueing system, where s_j beds are assigned to service j and s_{N+1} beds are assigned to the central pool. In the following analysis, advantage is taken of this characteristic.

Property 1. The state probabilities $P_j^*(m)$ for the $M/E_k/s_j/s_{N+1}^*$ queueing system retain the Poisson characteristic for states $m = 0, 1, 2, \dots, s_j$, i.e.,

$$P_j^*(m) = \begin{cases} P_j^*(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} & m=0,1,2,\dots,s_j \\ \text{unknown} & m=s_j+1,\dots,s_j+s_{N+1} \\ 0 & m > s_j+s_{N+1} \end{cases}$$

$P_j^*(0)$ cannot be defined precisely, since the availability of central pool beds depends on the overflow probabilities of other services in the hospital system. However, due to the fact that the relationships between the first s_j states are unchanged, it follows that the relative relationships between state probabilities are of the form

$$P_j^*(m+1) = P_j^*(m) \frac{\lambda}{\mu_j(m+1)} \quad m = 0, 1, 2, \dots, s_j - 1 .$$

Property 2. The sum of state probabilities over states $m = s_j + 1, \dots, s_j + s_{N+1}$ for the $M/E_k/s_j/s_{N+1}^*$ queueing system is bounded above by that of the $M/E_k/s_j/s_{N+1}$ queueing system, i.e.,

$$\sum_{m=s_j+1}^{s_j+s_{N+1}} p_j^*(m) \leq \sum_{m=s_j+1}^{s_j+s_{N+1}} p_j(m)$$

Proof. Consider the $M/E_k/s_j/s_{N+1}$ queueing system, whose state probabilities are described by the truncated Poisson distribution. It is assumed that servers $s_j + 1$ through $s_j + s_{N+1}$ are always available. In the $M/E_k/s_j/s_{N+1}^*$ system, servers $s_j + 1$ through $s_j + s_{N+1}$ are not necessarily always available when requested by an arriving customer. When a server is not available, the arriving customer is rejected from the system. It follows that the probability of having $s_j + 1$ through $s_j + s_{N+1}$ customers in the system is reduced from that of the $M/E_k/s_j/s_{N+1}$ system.

Q.E.D.

Property 3. The state probabilities $p_j^*(m)$ for the $M/E_k/s_j/s_{N+1}^*$ system are bounded below by the state probabilities $p_j(m)$ for the $M/E_k/s_j/s_{N+1}$ system for $m = 0, 1, 2, \dots, s_j$, i.e.,

$$p_j^*(m) \geq p_j(m) \quad m = 0, 1, 2, \dots, s_j .$$

Proof. Since state probabilities sum to 1.0, it follows from Properties 1 and 2 that the state probabilities 0 through s_j are no less for the $M/E_k/s_j/s_{N+1}^*$ system than the $M/E_k/s_j/s_{N+1}$ system.

Q.E.D.

Property 4. The state probabilities $p_j^*(m)$ for the $M/E_k/s_j/s_{N+1}^*$ queueing system are bounded above by the state probabilities $p_j(m)$ for $M/E_k/s_j/0$

queueing system for states $m = 0, 1, 2, \dots, s_j$, i.e.,

$$P_j^*(m) \leq P_j(m) \quad 0, 1, 2, \dots, s_j .$$

Proof.

$$\sum_{m=0}^{s_j} P_j^*(m) \leq 1$$

or

$$P_j^*(0) \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \leq 1$$

$$P_j^*(0) \leq \frac{1}{\sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!}}$$

therefore

$$P_j^*(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \leq \frac{1}{\sum_{n=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^n \frac{1}{n!}} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!}$$

Q.E.D.

Property 5. The state probability $P_j^*(s_j + s_{N+1})$ of the $M/E_k/s_j/s_{N+1}^*$ system is bounded from above by the state probability $P_j(s_j + s_{N+1})$ of the $M/E_k/s_j/s_{N+1}$ queueing system, i.e.,

$$P_j^*(s_j + s_{N+1}) \leq P_j(s_j + s_{N+1}) .$$

Proof. For the $M/E_k/s_j/s_{N+1}$ system, $s_j + s_{N+1}$ servers are continuously available for use. For the $M/E_k/s_j/s_{N+1}^*$, s_j servers are available on a random basis. Since for the $M/E_k/s_j/s_{N+1}^*$ system, there would be, on the average, less than $s_j + s_{N+1}$ total servers available, it follows that the probability of having the system full, (i.e., $s_j + s_{N+1}$ customers in the system) would be less than for the $M/E_k/s_j/s_{N+1}$ system.

Q.E.D.

By Property 3, the probability mass function for the $M/E_k/s_j/s_{N+1}^*$ system is point by point greater than that of the $M/E_k/s_j/s_{N+1}$ system for states $m=0,1,\dots,s_j$. By Property 2, the probability mass functions for the $M/E_k/s_j/s_{N+1}^*$ and the $M/E_k/s_j/s_{N+1}$ systems must cross at some point in region s_j, s_j+s_{N+1} . A sketch of the probability mass function for the $M/E_k/s_j/s_{N+1}^*$ queueing system in comparison to the probability mass functions for the $M/E_k/s_j/0$ and the $M/E_k/s_j/s_{N+1}$ systems is in Figure 3.5. Continuous curves are used to approximate the discrete probability mass functions.

3.3.3 Improving the Bounds on State Probabilities of the $M/E_k/s_j/s_{N+1}$ Queueing Systems

Let the states of the queueing systems be divided into two regions:

- Region 1 for states $m = 0,1,2,\dots,s_j$, which includes all states where service j does not use any beds other than its own, and
- Region 2 for states $m = s_j+1,\dots,s_j+s_{N+1}$, which includes all states where service j has to use beds in the central pool.

Consider an N -service hospital system. The lower and upper bounds on the state probabilities in Region 1 of a service can be obtained from Properties 2 and 3. The relationships between a single service and all the others can be used to tighten the bounds on the state probabilities for the single service $M/E_k/s_j/s_{N+1}^*$ queueing systems. For instance, the upper bound on the probability that any of the first $N-1$ services use the central pool can be used to derive a lower bound on the probability that the N^{th} service uses the central pool. Similarly, the lower bound on the probability that the first $N-1$ services use central pool beds can be used to derive a new upper bound on the probability that the N^{th} service uses

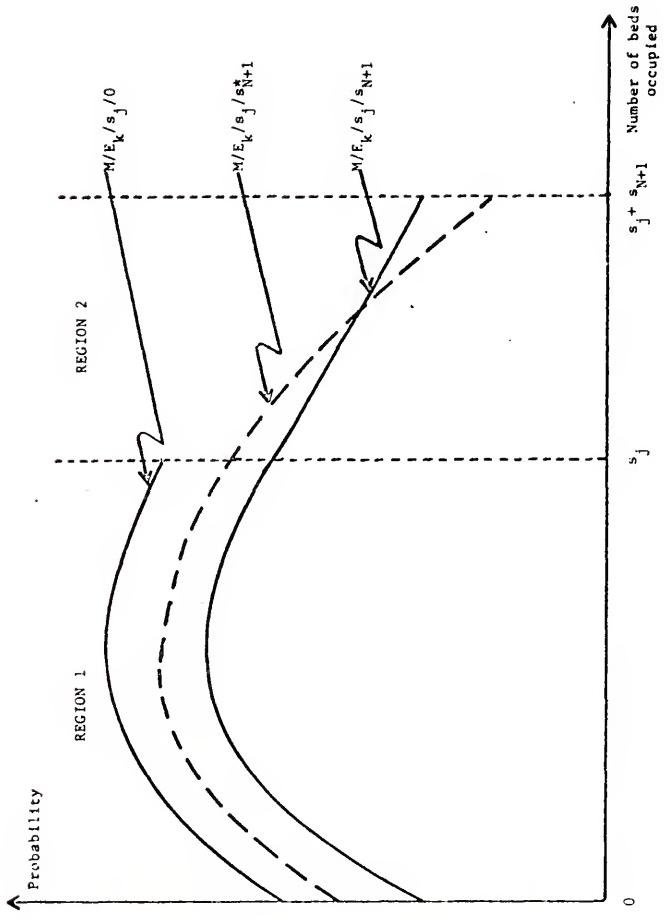


Figure 3.5. The bounds on the probability function of the $M/E_k/s_j/s_{N+1}^*$ queueing system.

central pool beds. The increased value of the lower bounds for central pool use decreases, in turn, the values of upper bounds of the state probabilities of Region 1 of each service. Continuing the procedure, the lower and upper bounds on the probability that a service uses central pool beds can be successively tightened, and the lower and upper bounds on the state probabilities in Region 1 can be improved accordingly. The properties necessary to support the above general procedure are now presented.

Let $\phi_j^*(m)$ be the probability of having m or more central pool beds available to service j , and ϕ_j^L, ϕ_j^U be the corresponding lower and upper bounds, respectively, i.e.,

$$\phi_j^U \geq \phi_j^*(m) \text{ for } m=1,2,\dots,s_{N+1}$$

$$\phi_j^L \leq \phi_j^*(m) \text{ for } m=1,2,\dots,s_{N+1}$$

Property 6. The lower and upper bounds, ϕ_j^L and ϕ_j^U , on the probability $\phi_j^*(m)$ for $m=1,2,\dots,s_{N+1}$ are

$\phi_j^L = \text{Lower bound } \{\text{probability that all the other services } (\neq j) \text{ use their own beds only}\} .$

$\phi_j^U = 1 - \sum_{i \neq j} \text{Lower bound } \{\text{probability that service } i \text{ uses all } s_{N+1} \text{ central pool beds}\}$

Proof. For service j , by definition

$\phi_j^*(m) = \text{the probability that there are } m \text{ or more central pool beds available for service } j$
 $= \text{the probability that other services } (\neq j) \text{ use no more than } (s_{N+1}-m) \text{ central pool beds}$

= 1 - the probability that other services ($\neq j$) use more than $(s_{N+1}-m)$ central pool beds.

Since the probability that other services ($\neq j$) use more than $(s_{N+1}-m)$ central pool beds is no greater than the probability that other services ($\neq j$) use any central pool beds, for $m=1, 2, \dots, s_{N+1}$, it follows that

$\phi_j^*(m) \geq 1 - \text{the probability that other services } (\neq j) \text{ use central pool beds.}$

Thus

$\phi_j^*(m) \geq 1 - \text{upper bound } \{ \text{the probability that other services } (\neq j) \text{ use central pool beds} \}$

or,

$\phi_j^*(m) \geq \text{lower bound } \{ \text{probability that other services } (\neq j) \text{ use their own beds} \}$

i.e.,

$$\phi_j^*(m) \geq \text{lower bound } \left\{ \prod_{i \neq j} \sum_{m=0}^{s_i} p_i^*(m) \right\}$$

$$\phi_j^*(m) \geq \prod_{i \neq j} \text{lower bound } \left\{ \sum_{m=0}^{s_i} p_i(m) \right\}$$

Also, the probability that other services ($\neq j$) use more than $(s_{N+1}-m)$ central pool beds is no less than the probability that other services ($\neq j$) use all central pool beds, for $m=1, 2, \dots, s_{N+1}$, it follows that

$\phi_j^*(m) \leq 1 - \text{probability that other services } (\neq j) \text{ use all central pool beds}$

thus

$\phi_j^*(m) \leq 1 - \sum_{i \neq j} \text{lower bound } \{ \text{the probability that service } i \text{ uses all central pool beds} \}$

i.e.,

$$\phi_j^*(m) \leq 1 - \sum_{i \neq j} \text{lower bound } \{ p_i^*(s_i + s_{N+1}) \} \quad \text{Q.E.D.}$$

Let the $M/E_k/s_j/s_{N+1}'$ queueing system be the system with two arrival rates

- Arrival Rate λ_j for states $m=0, 1, 2, \dots, s_j$.
- Arrival Rate $\lambda_j \phi_j^L$ for states $m=s_j+1, \dots, s_j+s_{N+1}$.

Similarly, define the $M/E_k/s_j/s_{N+1}''$ queueing system to be the system with two arrival rates λ_j and $\lambda_j \phi_j^U$.

Property 7. The upper bounds $p_j^U(m)$ on state probabilities in Region 1 of service j are

$$p_j^U(m) = p_j'(0) \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} \quad \text{for } m=0, 1, 2, \dots, s_j ,$$

where

$$p_j'(0) = \frac{1}{Z}$$

$$Z = \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} (\phi_j^L)^{m-s_j}$$

ϕ_j^L = the lower bound on the probability that m or more central pool beds are available for service j (Property 6), for $m=1, 2, \dots, s_{N+1}$.

Proof. Using the relationships between state probabilities $p_j^*(m)$ presented in Appendix A the total probability for the queueing system $M/E_k/s_j/s_{N+1}$ is

$$\sum_{m=0}^{s_j+s_{N+1}} p_j^*(m) = p_j^*(0) \left\{ \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} \prod_{n=1}^{m-s_j} \phi_j^*(n) \right\} .$$

Using the lower bound ϕ_j^L in place of $\phi_j^*(n)$, for all n , we have

$$\sum_{m=0}^{s_j+s_{N+1}} p_j^*(m) \geq p_j^*(0) \left\{ \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \left(\phi_j^L\right)^{m-s_j} \right\}.$$

Since the total probability is 1, we have

$$\frac{1}{\left\{ \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \left(\phi_j^L\right)^{m-s_j} \right\}} \geq p_j^*(0) .$$

Let Z denote the value within the brackets. Then

$$\frac{1}{Z} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \geq p_j^*(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \quad \text{for } m=0,1,2,\dots,s_{N+1}$$

Q.E.D.

Property 8. The lower bound on the sum of the state probabilities of those states where service j uses central pool beds can be found accordingly:

$$\sum_{n=s_j+1}^{s_j+s_{N+1}} p_j^*(n) \geq 1 - \sum_{n=0}^{s_j} p_j^*(0) \left(\frac{\lambda_j}{\mu_j}\right)^n \frac{1}{n!} .$$

Proof. Property 8 follows directly from Property 7.

Property 9. The lower bound on the state probability $p_j^*(s_j+s_{N+1})$ for service j is

$$p_j^L(s_j+s_{N+1}) = p_j^*(0) \left(\frac{\lambda_j}{\mu_j}\right)^{s_j+s_{N+1}} \frac{1}{(s_j+s_{N+1})!} \left(\phi_j^L\right)^{s_{N+1}} .$$

Proof. By definition, the probability ϕ_j^L that central pool beds will be available for patients of a service in the $M/E_k/s_j/s_{N+1}$ queueing system is the lower bound of the probability ϕ_j^* of that for the $M/E_k/s_j/s_{N+1}^*$ system. Thus, the probability of finding all central pool beds occupied by patients of a service for the $M/E_k/s_j/s_{N+1}$ queueing system is smaller than that

of the $M/E_k/s_j/s_{N+1}^*$ system, i.e.,

$$P_j^*(s_j+s_{N+1}) \geq P_j'(0) \left(\frac{\lambda_j}{\mu_j}\right)^{s_j+s_{N+1}} \frac{1}{(s_j+s_{N+1})!} (\phi_j^L)^{s_{N+1}} .$$

Q.E.D.

Property 10. The lower bound $P_j^L(m)$ on state probabilities in Region 1 of service j are

$$P_j^L(m) = P_j''(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \quad \text{for } m=0,1,2,\dots,s_j ,$$

where

$$P_j''(0) = \frac{1}{Z}$$

$$Z = \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} (\phi_j^u)^{m-s_j}$$

ϕ_j^u = the upper bound on the probability that m or more central pool beds are available for service j , from Property 6, for $m=1,2,\dots,s_{N+1}$.

Proof. The proof for Property 10 is similar to that for Property 7 with the $M/E_k/s_j/s_{N+1}''$ system in place of $M/E_k/s_j/s_{N+1}'$ and rates $\lambda_j, \lambda_j \phi_j^u$.

Property 11. The upper bound on the sum of the state probabilities of those states where service j uses central pool beds can be found accordingly:

$$\sum_{m=s_j+1}^{s_j+s_{N+1}} P_j^*(m) \leq 1 - \sum_{m=0}^{s_j} P_j''(0) \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} .$$

Proof. Property 11 follows directly from Property 10.

Property 12. The upper bound on the state probability $P_j^*(s_j+s_{N+1})$ for service j is

$$P_j^U(s_j+s_{N+1}) = P_j^U(0) \left(\frac{\lambda_j}{\mu_j}\right)^{s_j+s_{N+1}} \frac{1}{(s_j+s_{N+1})!} (\phi_j^U)^{s_{N+1}} .$$

Proof. The proof for Property 12 is similar to that of Property 9.

Property 13. The lower bounds on the state probabilities in Region 2 of service j are

$$P_j^L(m) = \text{Max} \left\{ P_j^L(s_j) \left(\frac{\lambda_j \phi_j^L}{\mu_j}\right)^{m-s_j} \frac{s_j!}{m!}, P_j^L(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^U}\right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \right\} .$$

Proof. Using the relationships developed in Appendix A and the upper bound probability ϕ_j^U , we have

$$\frac{P_j^*(m+1)}{P_j^*(m)} \leq \frac{\lambda_j \phi_j^U}{\mu_j(m+1)} \quad \text{for } m=s_j+1, \dots, s_j+s_{N+1}-1 .$$

Therefore

$$\frac{P_j^*(s_j+s_{N+1})}{P_j^*(m)} \leq \left(\frac{\lambda_j \phi_j^U}{\mu_j}\right)^{s_j+s_{N+1}-m} \frac{m!}{(s_j+s_{N+1})!} .$$

Using the lower bound probability, $P_j^L(s_j+s_{N+1})$, we obtain

$$P_j^L(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^U}\right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \leq P_j^*(m)$$

Similarly

$$\frac{P_j^*(m)}{P_j^*(m-1)} \geq \frac{\lambda_j \phi_j^L}{\mu_j} \quad \text{for } m=s_j+1, \dots, s_j+s_{N+1} .$$

Thus

$$\frac{P_j^*(m)}{P_j^*(s_j)} \geq \left(\frac{\lambda_j \phi_j^L}{\mu_j}\right)^{m-s_j} \frac{s_j!}{m!}$$

Using the lower bound probability $p_j^L(s_j)$, we obtain

$$p_j^*(m) \geq p_j^L(s_j) \left(\frac{\lambda_j \phi_j^L}{\mu_j} \right)^{m-s_j} \frac{s_j!}{m!}$$

Therefore

$$p_j^*(m) \geq \text{Max} \left\{ p_j^L(s_j) \left(\frac{\lambda_j \phi_j^L}{\mu_j} \right)^{m-s_j} \frac{s_j!}{m!}, p_j^L(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^U} \right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \right\}$$

$$\text{for } m=s_j+1, \dots, s_j+s_{N+1}.$$

Intuitively, the lower bound probabilities, $p_j^L(m)$, are found by decreasing the probabilities with a higher rate from a lower bound state probability at s_j , or by increasing at a lower rate toward s_j from a lower bound state probability at s_j+s_{N+1} . This method is illustrated in Figure 3.6.

Property 14. The upper bounds on state probabilities in Region 2 of service j are

$$p_j^U(m) = \text{Min} \left\{ p_j^U(s_j) \left(\frac{\lambda_j \phi_j^L}{\mu_j} \right)^{m-s_j} \frac{s_j!}{m!}, p_j^U(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^L} \right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \right\}$$

Proof. The proof for this property is similar to that of Property 13.

Intuitively, the upper bound probabilities $p_j^U(m)$ are found by decreasing the probabilities at a lower rate from an upper bound state probability at s_j , or by increasing at a higher rate toward s_j from an upper bound state probability at s_j+s_{N+1} . This method is illustrated in Figure 3.7.

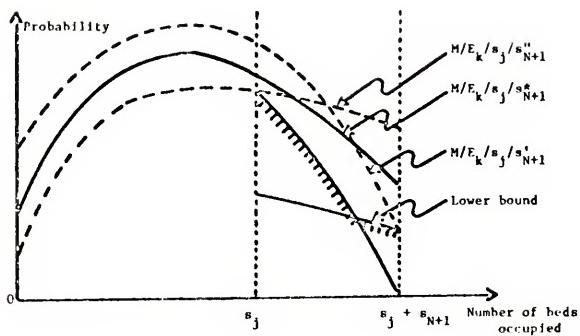


Figure 3.6. The lower bounds on the state probability for the $M/E_k/s_j/s_{N+1}^*$ queueing system.

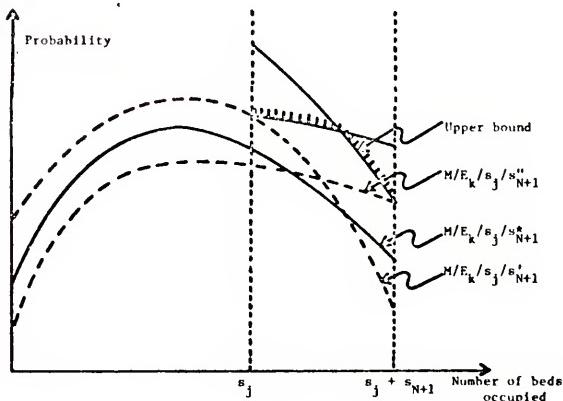


Figure 3.7. The upper bounds on the state probability for the $M/E_k/s_j/s_{N+1}^*$ queueing system.

3.3.4 An Algorithm for Evaluating the State Probability Bounds

Using the above properties, the upper and lower bounds on the state probabilities over Regions 1 and 2 can be successively tightened. Thus, good approximation of the state probabilities of the $M/M/s_j/s_{N+1}^*$ system can be obtained. The following algorithm summarizes the steps of the procedure.

Step 1. Evaluate the state probabilities (the truncated Poisson probabilities) for the $M/E_k/s_j/s_{N+1}$ queueing system.

$$P_j(m) = \frac{1}{\sum_{i=0}^{s_j+s_{N+1}} \frac{\lambda_j^i}{\mu_j^i} i!} \frac{(\lambda_j)^m}{\mu_j^m} \frac{1}{m!}$$

for $m=0,1,2,\dots,s_j+s_{N+1}$, and $j=1,2,\dots,N$.

Step 2. Evaluate the lower bounds on the probability that a service uses m beds of its own, (Property 3)

$$P_j^L(m) = P_j(m)$$

for $m = 0,1,2,\dots,s_j$ and $j = 1,2,\dots,N$.

Step 3. Evaluate the lower bound on the probability that m or more central pool beds are available for a service, (Property 6), for $m=1,2,\dots,s_{N+1}$

$$\phi_j^L = \prod_{i \neq j} \prod_{n=0}^{s_i} P_i^L(n)$$

Step 4. Improve the upper bounds on state probabilities in Region 1, (Property 7)

$$P_j^U(m) = P_j'(0) \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} \quad \text{for } m = 0,1,2,\dots,s_j, \text{ and} \\ \text{for } j = 1,2,\dots,N,$$

where

$$P_j'(0) = \frac{1}{\left\{ \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} (\phi_j^L)^{m-s_j} \right\}}.$$

Step 5. Improve the lower bound $P_j^L(s_j+s_{N+1})$ on the state probability (Property 9):

$$P_j^L(s_j+s_{N+1}) = P_j'(0) \left(\frac{\lambda_j}{\mu_j} \right)^{s_j+s_{N+1}} \frac{1}{(s_j+s_{N+1})!} (\phi_j^L)^{s_{N+1}}.$$

Step 6. Evaluate the upper bound on the probability that m or more central pool beds are available for a service, (Property 6), for $m=1, 2, \dots, s_{N+1}$:

$$\phi_j^U = 1 - \sum_{i \neq j} P_i^L(s_i+s_{N+1})$$

Step 7. Improve the lower bounds on state probabilities in Region 1, (Property 10).

$$P_j^L(m) = P_j''(0) \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} \quad \text{for } m=0, 1, 2, \dots, s_j, \text{ and} \\ \text{for } j=1, 2, \dots, N$$

where

$$P_j''(0) = \frac{1}{\left\{ \sum_{m=0}^{s_j} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} + \sum_{m=s_j+1}^{s_j+s_{N+1}} \left(\frac{\lambda_j}{\mu_j} \right)^m \frac{1}{m!} (\phi_j^U)^{m-s_j} \right\}}$$

Step 8. Test for improvement on the lower and upper bounds on the probabilities over Region 1. If improvements are less than some Epsilon, for all states $m=0, 1, \dots, s_j$ then go to Step 9; otherwise, go to Step 3.

Step 9. Evaluate the lower bounds on the state probabilities in Region 2
(Property 13)

$$P_j^L(m) = \text{Max} \left\{ P_j^L(s_j) \left(\frac{\lambda_j \phi_j^L}{\mu_j} \right)^{m-s_j} \frac{s_j!}{m!}, P_j^L(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^U} \right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \right\}$$

for $m=s_j+1, \dots, s_j+s_{N+1}$, and

for $j=1, 2, \dots, N$.

Step 10. Evaluate the upper bounds on the state probabilities in Region 2
(Property 14)

$$P_j^U(m) = \text{Min} \left\{ P_j^U(s_j) \left(\frac{\lambda_j \phi_j^U}{\mu_j} \right)^{m-s_j} \frac{s_j!}{m!}, P_j^U(s_j+s_{N+1}) \left(\frac{\mu_j}{\lambda_j \phi_j^U} \right)^{s_j+s_{N+1}-m} \frac{(s_j+s_{N+1})!}{m!} \right\}$$

for $m=s_j+1, \dots, s_j+s_{N+1}$, and

for $j=1, 2, \dots, N$.

Stop.

3.3.5 Verification of the Bounds on the State Probabilities

A simulation model was used to evaluate the state probabilities for the $M/E_k/s_j/s_i^*$ queueing system. A comparison of the analytic results and the simulation results was made to evaluate the usefulness of the analytic bounds on the state probabilities.

3.3.5.1 A Simulation Model of a Three-Service Hospital

A simulation model of a three-service hospital was constructed using the GPSS language under the following assumptions:

- The arrival process is Poisson distributed.
- The patient length of stay is distributed as an Erlang-2 distribution.

- A central pool with beds which can be used by overflow patients of any of these three services.
- Patients in central pool beds have to return to their original service beds as soon as there are available beds.

The flow chart of the simulation model is presented in Appendix B.

3.3.5.2 Comparison of the Analytic and Simulation Results

An example of a three-service hospital system with a central pool is used. The parameters for the system are in Table 3.1. The output of the bounds on state probabilities of the analytic results is plotted against the state residence time distribution function obtained from the simulation results in Figures 3.8, 3.9, and 3.10.

The simulation was run for a simulated period of seven years, before a state residence time distribution function was obtained that was within the analytic bounds. The cost in CPU time of the simulation is 569 seconds on an IBM-370-165. The analytic bounds on the state probabilities were obtained using a FORTRAN program which ran in 1.06 seconds. The agreement between the simulation and analytic provides additional assurance that the procedure for determining the analytic bounds on the state probabilities are of value in evaluating the system operating characteristics. Moreover, for a given level of accuracy, finding the analytic bounds cost considerably less than the simulation results. The costs for obtaining the analytic bounds increase slowly as shown in Table 3.2.

3.3.6 Analogous Analysis for Models 2 and 4

For Model 2, the analysis for the bounds on state probabilities of the N-service hospital is similar to that of Model 3. The probability that a service uses its own beds can be used to improve the bounds on

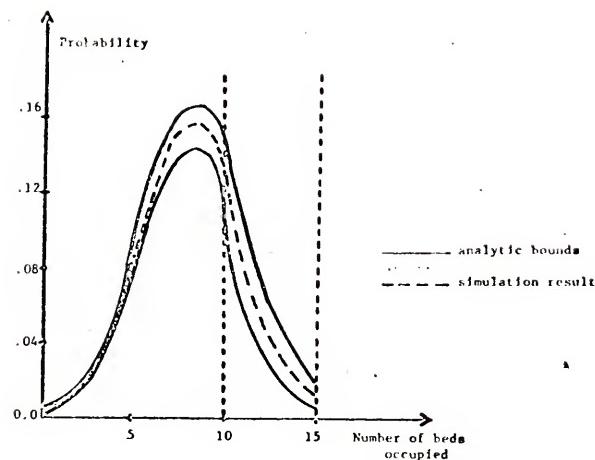


Figure 3.8. The state probabilities and bounds for service 1.

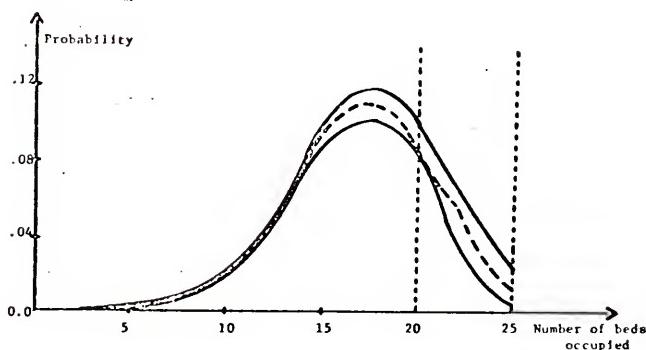


Figure 3.9. The state probabilities and bounds for service 2.

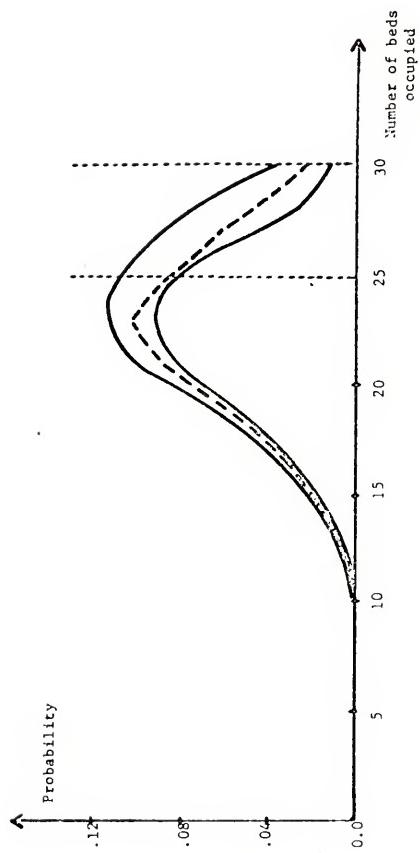


Figure 3.10. The state probabilities and bounds for service 3.

Table 3.1

Parameters for a Three-Service Hospital System
Used in the Example for State Probability Bounds.

Service	Number of beds	Mean arrival rate (patients/day)	Length of stay (days)
1	10	2	4.5
2	20	3	6.0
3	25	3	8.0
Central pool	5		

Table 3.2
Computational Costs for Analytic Results.
(Stopping Criteria $\epsilon = 10^{-7}$)

Number of services*	Number of beds in central pool	CPU time in seconds
5	5	1.19
6	5	1.23
7	10	1.34
9	10	2.37

*Each service has 25 beds.

state probabilities of other services. The upper bound on the probability that a service uses m or less beds of its own can be used to derive the lower bound on the probability that other services use $(B-m)$ beds. Similarly, the lower bound on the probability that a service uses m or less beds of its own can be used to derive the upper bound on the probability that other services use $(B-m)$ beds. These relationships between the N services can then be used successively to tighten the bounds on the state probabilities.

For Model 4, the analysis is again analogous to the one presented for Model 3. The two known probability distributions used as bounds on the state probabilities are those of the $M/E_k/s_j/s_{N+1}$ and $M/E_k/s_j/(B-s_j)$ queueing systems, where $j=1,2,\dots,N$. The bounds on the probability that a service has m or less beds occupied ($0 \leq m \leq s_j+s_{N+1}$) can be used to improve the bounds on the probability that other services have $(B-m)$ beds occupied.

3.4 Solution Techniques for the Bed Allocation Problem

The allocation problem for Models 2, 3 and 4 can be written generally as

Min (costs of allocating B beds among N services)

$$\text{s.t. } \sum_{j=1}^{N+1} s_j = B \quad (\text{for a system with a central bed pool})$$

$$\text{or } \sum_{j=1}^N s_j = B \quad (\text{for a system without central bed pool})$$

The expected costs of the allocation problem are not any kind of special functions such as linear or convex. The constraints are linear,

therefore, the problem can be solved by using a heuristic algorithm that searches for the minimum costs.

Let a bed allocation to be denoted as $n(s_1, s_2, \dots, s_N)$ where s_i beds are assigned to service i .

Definition: A neighboring point of an allocation (s_1, s_2, \dots, s_N) is any allocation which can be presented by a perturbation of the above allocation $(s_1 + a_1, s_2 + a_2, \dots, s_N + a_N)$ such that

$$\sum_{i=1}^N a_i = 0$$

and $a_k = 1, a_1 = -1$ for $k \neq 1$

$a_m = 0$ for all $m \neq k, 1$

Therefore, there are at most $N(N-1)$ neighboring points for each allocation.

3.4.1 Heuristic Algorithm for Solving the Bed Allocation Problem

Step 1. Choose a set of (s_j) that satisfies the constraint.

Let this allocation be n^0 .

Step 2. Find the objective function value, $f(n^0)$, for the allocation n^0 .

Step 3. Generate all neighboring points of allocation n^0 .

Step 4. Find objective function values $f(n_j^0)$ for all neighboring points of allocation n^0 .

Step 5. Take the minimum of objective values.

If $f(n_x^0) = \text{Min } f(n_j^0) \leq f(n^0)$, then let $n^0 = n_x^0$ go to

Step 2. Otherwise, $n^* = n^0$, where n^* is a local optimum solution of the allocation problem; stop.

The allocation problem is solved twice, once for the upper bounds on state probabilities and once for the lower bounds on state probabilities. The allocations of the two problems are compared, if they are equal the heuristic allocation of the problem is found. The approximate allocation is considered acceptable when the two allocations differ by one or two beds.

3.4.2 Experimental Results for the Bed Allocation Problem for Model 3

An example of the bed allocation problem for Model 3 is shown for a hospital with two services and a central bed pool using the procedures presented in previous sections. The parameters for the system are in Table 3.3.

The computational results are presented in Table 3.4. The allocations were found after 5 iterations to be the same for the lower and upper bounds on the state probabilities. Therefore, the allocation (30, 19, 1) is the best allocation found for the system.

3.5 The Relative Costs in Bed Allocation Models

The costs of having a patient in a borrowed bed, in a central pool bed or turned away are difficult to evaluate in monetary values. Singh (61) attempted to evaluate the shortage costs by considering both the costs to the hospital and the costs to the patient. The absolute costs to the patient vary with individual cases and depend on many factors such as medical condition, social and economical inconvenience. Singh surveyed some sample patients at various economic levels and constructed utility functions to determine the costs of admission delays to the patient. In this study, the objective functions of the models only require relative costs between services, elaboration on the determination of the cost as in Singh's study is not necessary. The emphasis is on the relative costs to

Table 3.3

Parameters for a Three-Service Hospital System
Used in an Example of Model 3

Service	Number of beds	Arrival rate (patients/day)	Length of Stay (days)	Central pool cost	Turnaway cost
1		2.15	5.81	6.11	40.16
2		2.24	8.93	1.03	30.36
3		1.92	6.35	2.10	60.73
Central pool	60				

Table 3.4
Computational Results for an Example of Model 3

Iteration	Number of beds				Expected Costs	
	Service 1	Service 2	Service 3	Central pool	Lower Bound	Upper Bound
1	15	20	20	5	21.28	21.56
2	16	19	20	5	20.21	20.78
3	17	18	20	5	19.63	20.35
4	18	17	20	5	19.53	20.21
5	19	17	19	5	19.32	19.80
6	18	18	19	5	19.03	19.60
7	18	19	18	5	18.93	19.30
8	18	20	18	4	18.69	19.71
9	17	21	18	4	18.60	19.67
10	18	21	18	3	18.43	19.60
11	17	22	18	3	18.24	19.49

- Other initial allocations attempted: (20, 15, 20, 5), (25, 15, 15, 5), and (20, 25, 13, 2).

the hospital and to the physicians for causing inconvenience to their patients and their medical practice. The method for evaluating the relative values of the costs is presented in the following section. The opinions of the health center officials at each institution can be used to validate these values. The Gainesville Veterans Administration Hospital was used as a sample for evaluating the relative costs.

3.5.1 The Method for Evaluating the Relative Costs

The characteristics of the existing allocation of the institution has to be identified: the number of services, the bed assignment of each service, and the interaction among services. One of the models presented in Section 3.2 can be chosen to describe the allocation of the system and the probability distribution functions of the number of beds occupied for each service can be determined by the method presented in Section 3.3. The expected numbers of patients in borrowed beds, in central pool beds, and turned away can be evaluated from the probability distribution functions.

The relative values of the costs are based on the corresponding expected numbers and the weight of importance for each service. For example the relative cost for bed borrowing for service j is found as the ratio of the weight of importance of service j and the expected number of borrowed beds service j . In this study, the number of beds assigned to each service is taken as the weight of importance for the service.

The relative values of the costs can be used to evaluate the cost for the existing allocation. The relative values of the costs are verified when the costs for the existing allocation are found to be the minimum costs with respect to any neighboring allocation.

3.5.2 Implementation Test

The Gainesville Veterans Administration Hospital was chosen for the test of evaluating the relative costs. A questionnaire was given to members of the hospital administrative and medical staffs to help in identifying the characteristics of the existing allocation system. These opinions of the officials were drawn from the following questions:

1. How many patients from other services can your unit absorb without impairing care to patients from your service or seriously inconveniencing your staff, given that beds are available?

2. How many patients assigned to your service can be placed in beds of other services before patient care is impaired or your staff is seriously inconvenienced?

3. What is your preference of the allocation of the off-service beds?

4. How many times per month can your service tolerate the situation where an emergency admission request causes a special action such as discharging a current patient early, holding of the new patient in a nonstandard area, or referring the patient to another service?

From the replies of the V.A. Hospital officials to the questionnaires, the existing allocation and its interaction among services are as follows. The hospital has eleven services grouped into four main services: Psychiatry, Medicine, Neurology, and Surgery. Psychiatry is completely segregated from other services; there is no interaction between Psychiatry and any other service in the hospital. Surgery is further divided into sub-services such as General Surgery, Thoracic Surgery, Plastic Surgery, Otolaryngology (E.N.T.), Urology, Neurosurgery, Orthopedics, and Ophthalmology. Medicine includes three sub-services: General Medicine, Pulmonary, and Cardiology. The Pulmonary and Cardiology services have been assigned a fixed number of beds

for their own patients recently. However, Pulmonary and Cardiology services still do not operate independently from the General Medicine services. In other words, medical services can freely borrow beds from one another without impairing care to patients. General Medicine, Pulmonary, and Cardiology therefore can be considered together as one service. It is totally unacceptable to the hospital officials to have medical patients placed in beds belonging to the surgical services. The same situation applies to the surgical services for having patients on medical floors. Thus, there is no interaction between medical and surgical services within the V.A. hospital. For the Gainesville Veterans Administration Hospital, it also appears that each of the four main services of the hospital have not faced a situation where an emergency patient is turned away due to no beds available.

In the following analysis, Psychiatry service is considered separately. The remaining system consists of three main services: Medicine, Neurology, and Surgery. In order to have some preliminary knowledge of the behavior of these services, it is assumed at first that there exists no interaction between these services. The $M/G/s_j/0$ queueing system provides the probability distribution of the number of beds occupied for each service. The distribution functions for Medicine, Neurology, and Surgery are plotted in Figures 3.11, 3.12, and 3.13 respectively. It can be easily seen from Figures 3.11 and 3.13 that Medicine and Surgery services have no chance of having more patients than their assigned numbers of beds. This result agrees with the replies from Medicine and Surgery services that there have been no problems in turning away patients for the last five years. Neurology has a highest chance of overflowing, however Neurology can send its patients to

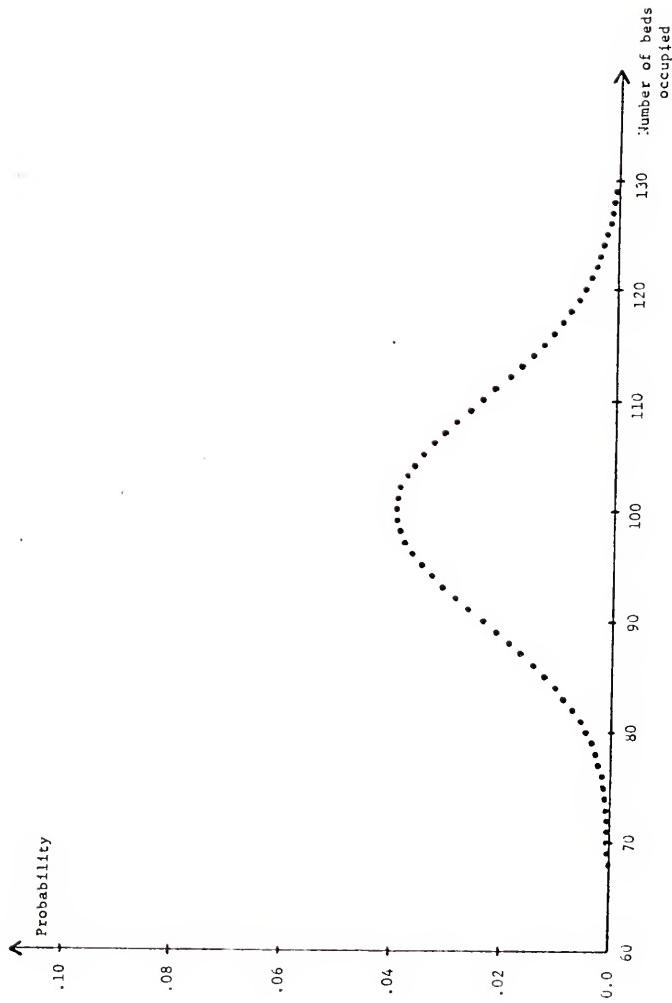


Figure 3.11. The Probability distribution of the number of beds occupied for Medicine service, Gainesville Veterans Administration Hospital.

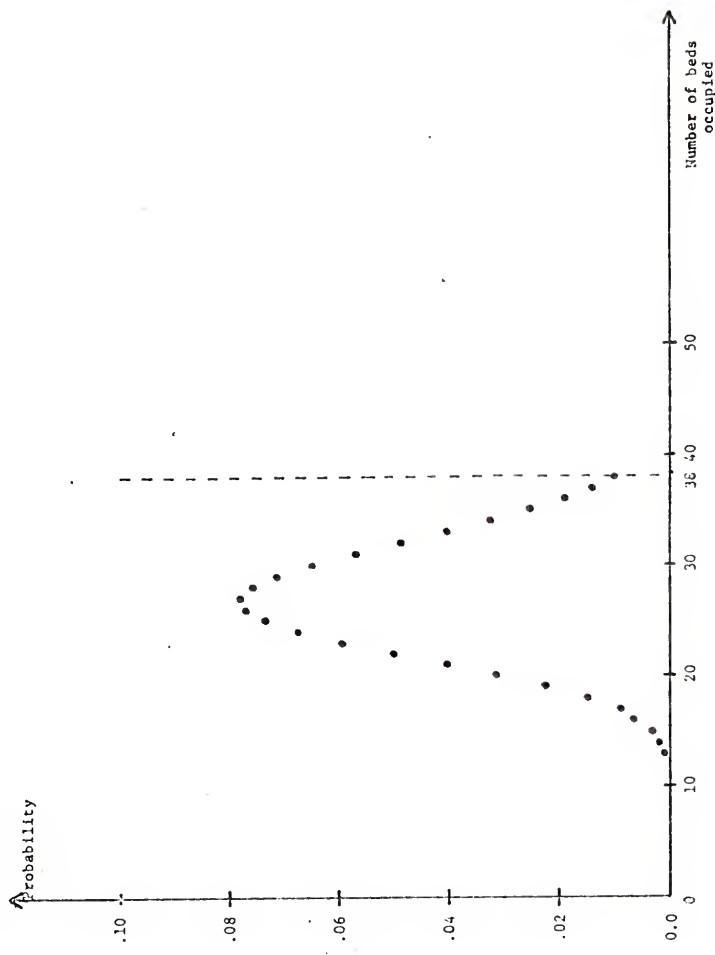


Figure 3.12. The probability distribution of the number of beds occupied for Neurology Service, Gainesville Veterans Administration Hospital.
(No interaction between services.)

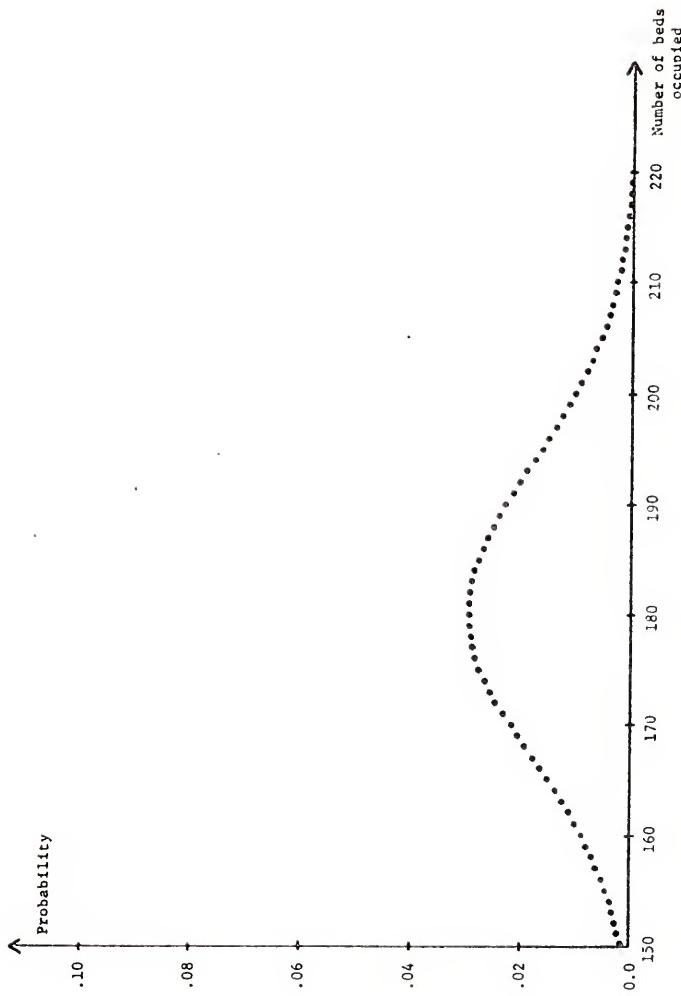


Figure 3.13. The probability distribution of the number of beds occupied for [redacted] Surgery service, Gainesville Veterans Administration Hospital.

off-service beds with the most preferable ones on Neurosurgery Ward. From these observations, the V.A. hospital system can be decomposed further: Medicine as a separate service, Neurology and Surgery services can be included in one system. A sketch of the decomposition of the system is illustrated in Figure 3.14.

Neurology and Surgery services are assigned a fixed number of beds to admit their own patients. If these services are considered independently, they can experience some overflow problems. Since these services are allowed to interact, that is borrow beds from one another, the bed capacity of each service can expand beyond its allocation. There is no chance of having a patient turned away; therefore, the system for an individual service is equivalent to the M/G/ ∞ queueing system. The probability that a service j has m beds occupied by its patients is the well-known Poisson distribution:

$$P_j(m) = e^{-\frac{\lambda_j}{\mu_j}} \left(\frac{\lambda_j}{\mu_j}\right)^m \frac{1}{m!} \quad \text{for } m=0,1,\dots$$

where λ_j = the mean arrival rate for service j , and

$\frac{1}{\mu_j}$ = the mean length of stay.

The probability distribution functions for services Neurology, ENT, General Surgery, Neurosurgery, Ophthalmology, Orthopedics, Plastic Surgery, Thoracic Surgery, and Urology are plotted in Figures 3.15, 3.16, 3.17, 3.18, 3.19, 3.20, 3.21, 3.22, and 3.23, respectively.

The number of beds occupied for each service is divided into two regions. Region 1 consists of all beds which are assigned to the

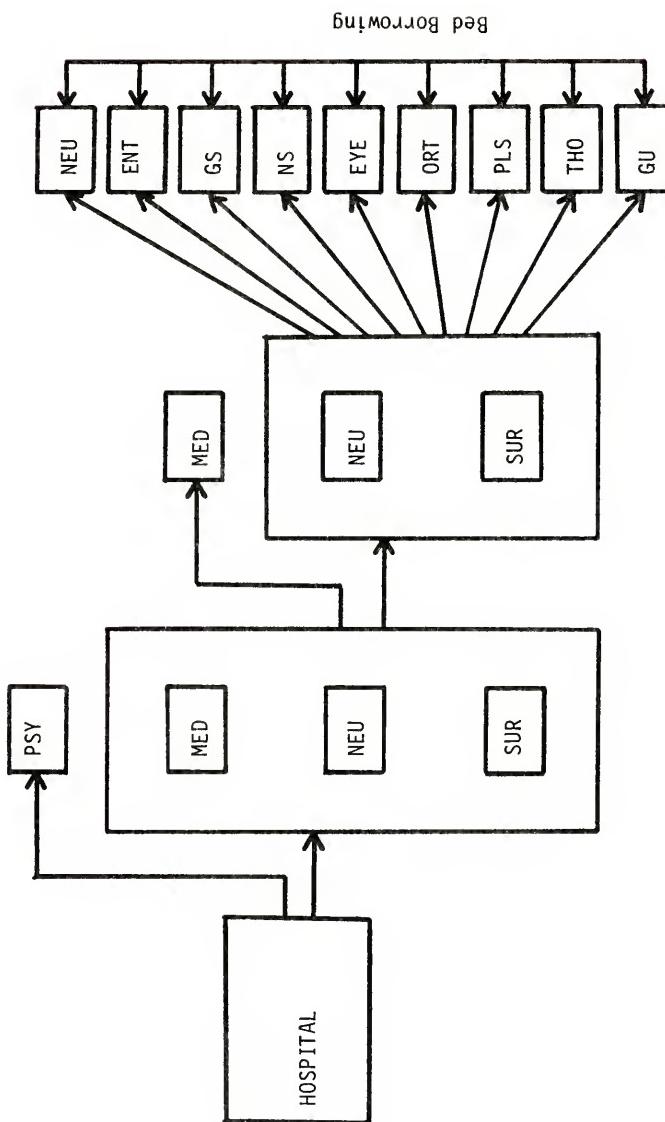


Figure 3.14 The decomposition of the V.A. Hospital system.

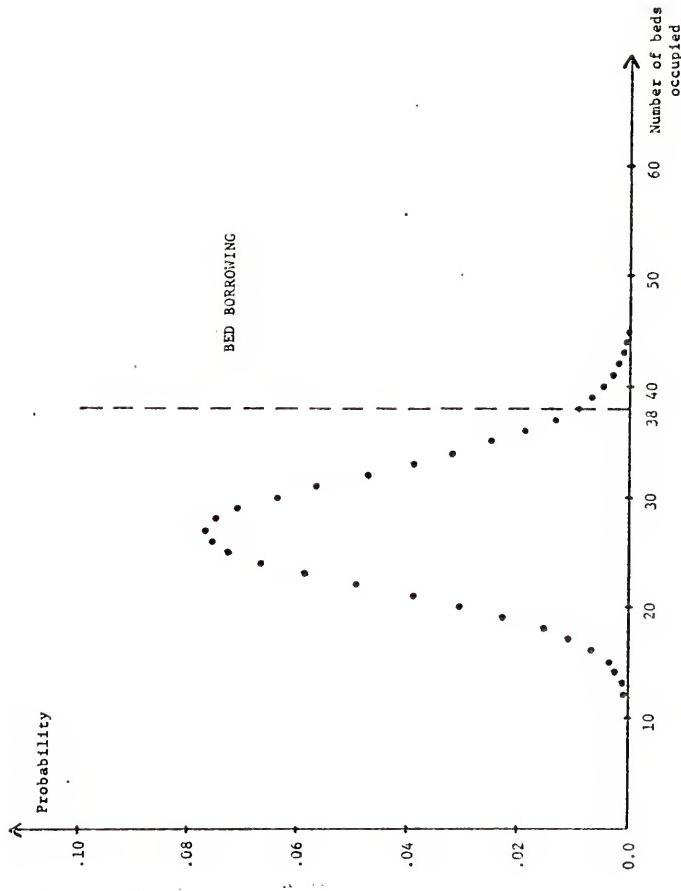


Figure 3.15. The probability distribution of the number of beds occupied for Neurology service, Gainesville Veterans Administration Hospital.

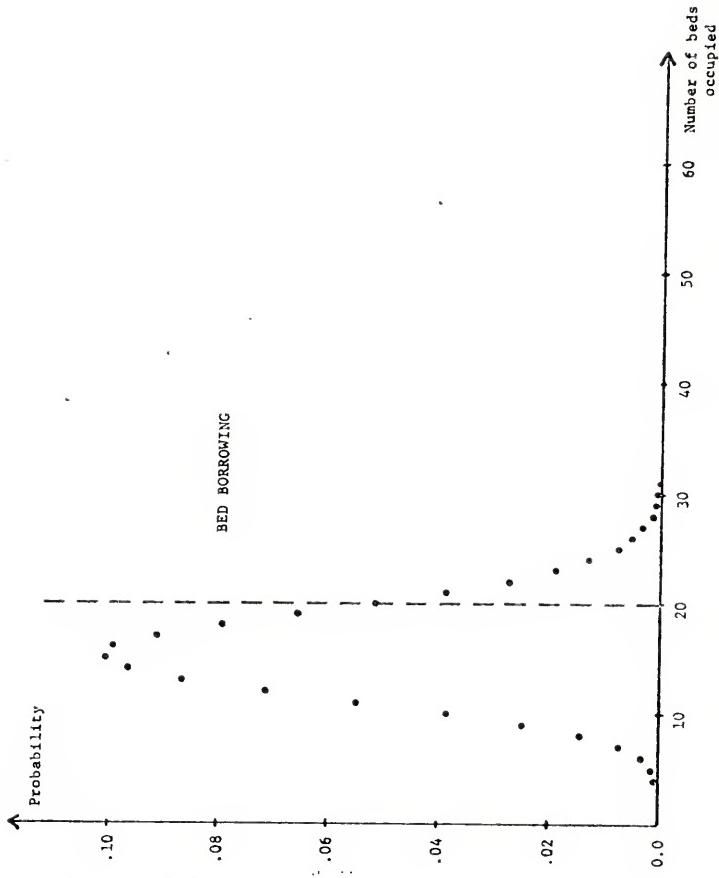


Figure 3.16. The probability distribution of the number of beds occupied for E.N.T. service, Gainesville Veterans Administration.

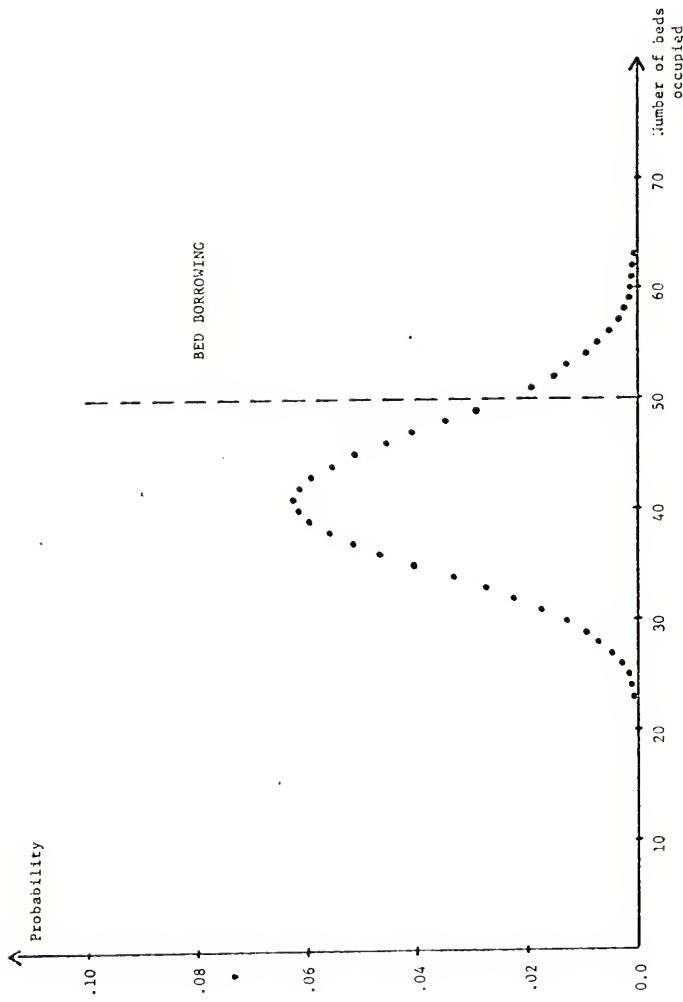


Figure 3.17. The probability distribution of the number of beds occupied for General Surgery service, Gainesville Veterans Administration Hospital.

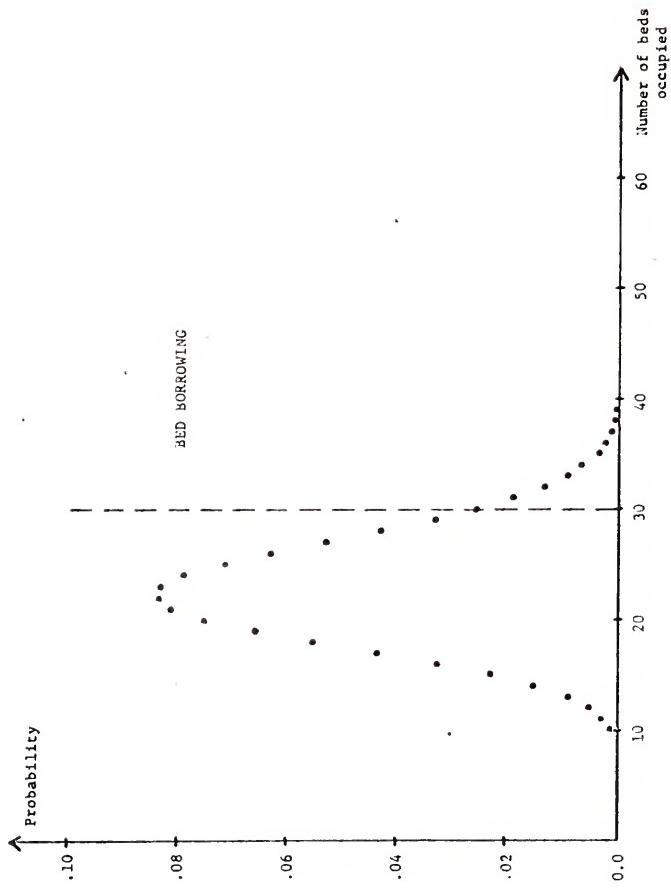


Figure 3.18. The probability distribution of the number of beds occupied for Neurosurgery service, Gainesville Veterans Administration Hospital.

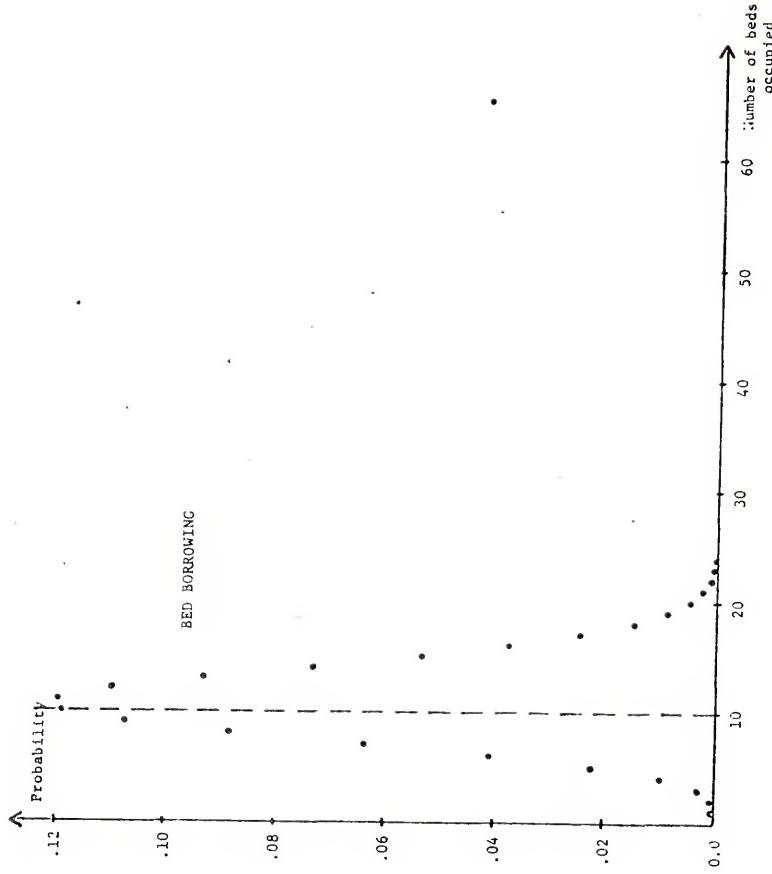


Figure 3.19. The probability distribution of the number of beds occupied for Bed Borrowing service, Gainesville Veterans Administration Service.

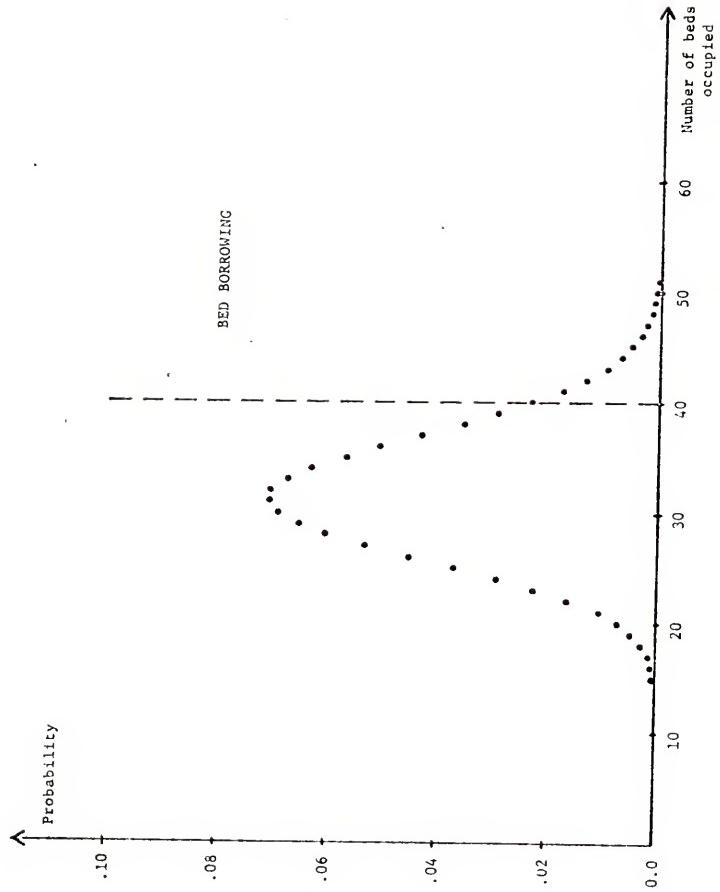


Figure 3.20. The probability distribution of the number of beds occupied for Orthopedics service, Gainesville Veterans Administration Hospital.

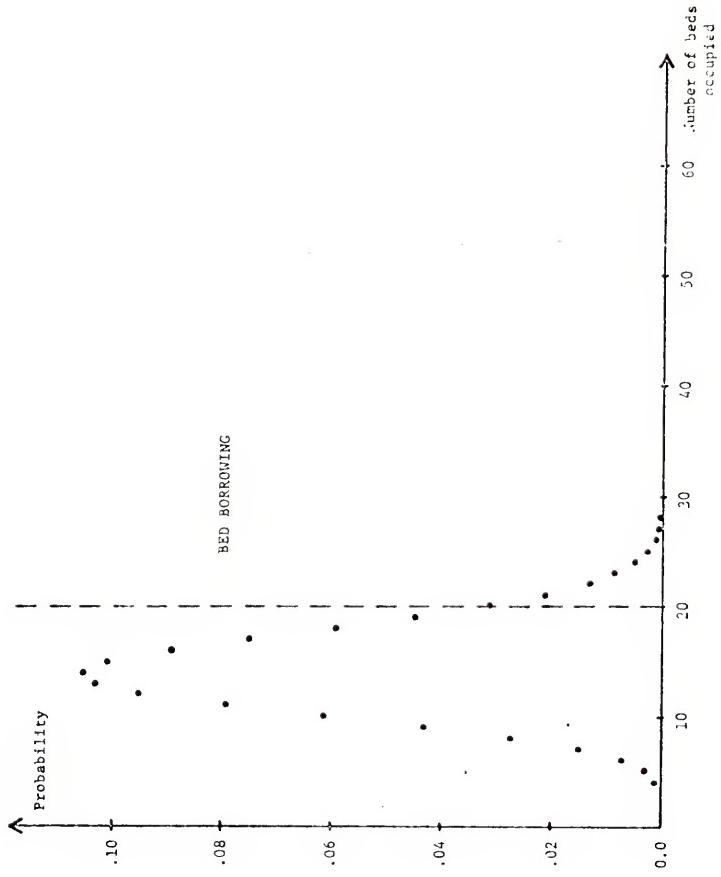


Figure 3.21. The probability distribution of the number of beds occupied for Plastic Surgery service, Gainesville Veterans Administration Hospital.

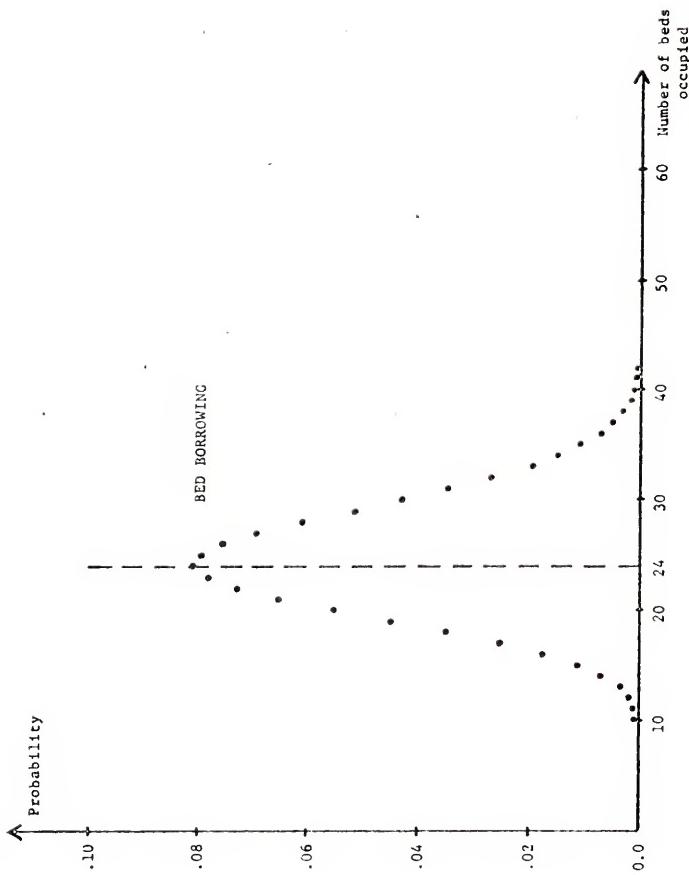


Figure 3.22. The probability distribution of the number of beds occupied for Thoracic Surgery service, Gainesville Veterans Administration Hospital.

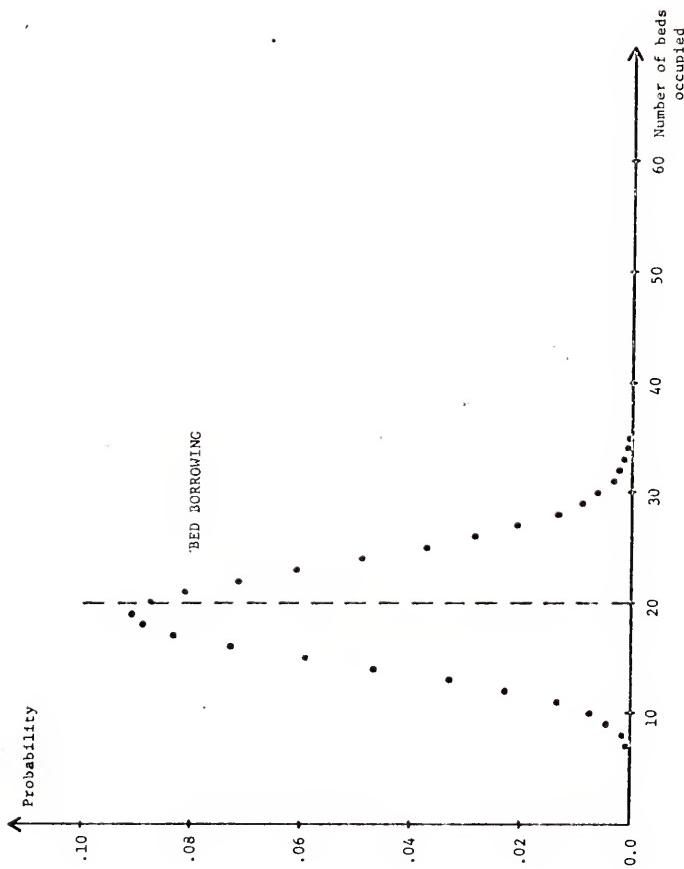


Figure 3.23. The probability distribution of the number of beds occupied for [Urology service, Gainesville Veterans Administration Hospital].

service, i.e., for $m=0,1,2,\dots,s_j$. Region 2 includes all beds that service j borrows from other services, i.e., for $m>s_j$. The expected rate of bed borrowing is equivalent to the rate of overflow in this case and can be found as

$$\lambda_j [1 - \sum_{m=0}^{s_j-1} p_j(m)] .$$

The expected number of borrowed beds for service j is the product of the expected rate of bed borrowing and the mean length of stay for patients of service j :

$$\frac{\lambda_j}{\mu_j} [1 - \sum_{m=0}^{s_j-1} p_j(m)] .$$

The relative costs of bed borrowing can be determined as the ratio of the number of beds assigned to service j and the expected number of borrowed beds for service j :

$$v_j = \frac{s_j}{\frac{\lambda_j}{\mu_j} [1 - \sum_{m=0}^{s_j-1} p_j(m)]}$$

The relative costs of bed borrowing for Neurology and Surgery services are listed in Table 3.5. The relative cost for Ophthalmology is smallest, 1.368, while the relative cost for Neurology is highest, 46,433. This can be interpreted to mean that the Neurology service is much more sensitive than Ophthalmology to having its patients in off-service beds. A Neurology patient in a borrowed bed is perceived to "cost" 33 times more than one for Ophthalmology. This can be easily seen from the current allocation where Neurology is separated from all surgical

Table 3.5
Relative Costs of Bed Borrowing

Services	Number of assigned beds	Expected number of beds borrowed	Relative costs
Neurology	38	.818	46.433
E.N.T.	20	2.611	7.659
General Surgery	50	4.259	11.038
Neurosurgery	30	1.867	16.070
Ophthalmology	10	7.308	1.368
Orthopedics	40	2.740	14.598
Plastic Surgery	20	1.246	16.050
Thoracic Surgery	24	14.419	1.664
Urology	20	9.175	2.180
Total	252		

services and Ophthalmology is one of the surgical services. The bed borrowing occurs more frequently and is more acceptable among the surgical services. The total costs of bed borrowing of the existing allocation can be determined by using the objective function of Model 2 presented in Section 3.2.2.

$$\text{Total costs} = \sum_{j=1}^9 v_j \lambda_j [1 - \sum_{m=0}^{s_j-1} p_j(m)] .$$

The relative costs can be verified by assuming the existing allocation is the best allocation for the system. Using the replies of hospital officials to question number 2, the acceptable number of borrowed beds for each service can be used as constraints on the allocation. The bed allocation for Model 2 can be modified as

$$\text{Min } \sum_{j=1}^9 v_j \lambda_j [1 - \sum_{m=0}^{x_j-1} p_j(m)]$$

$$\text{s.t. } \sum_{j=1}^9 x_j = 252$$

$$\frac{\lambda_j}{v_j} [1 - \sum_{m=0}^{x_j-1} p_j(m)] \leq a_j$$

$$x_j \text{ integer, } j = 1, 2, \dots, 9$$

where a_j is the acceptable number of borrowed beds for service j .

It should be noted that the above optimization problem may not have a feasible solution. The existing allocation is assumed to be the allocation with minimum costs. The costs for neighboring allocations which do not violate the constraints are evaluated using the relative costs v_j 's. The costs for these neighboring allocations are compared to those of the

existing allocation as in Table 3.6. The costs for the existing allocation are found to be minimum with the relative costs V_j 's. In other words, using relative costs V_j 's found previously, there is no allocation which provides lower costs than the existing one without violating the constraints. Thus, the relative costs V_j 's can be accepted as the cost values of the existing allocation for the Gainesville Veterans Administration Hospital.

For the cases where the relative costs V_j 's yield a minimum cost allocation other than the existing one, a review of the questionnaire replies with the hospital officials should take place to decide on new values of constraints, or on the characteristics of the existing allocation.

Table 3.6

The Costs for the Existing Allocation
and Its Neighboring Allocations

Allocation*	Costs
Existing	19.36
37 beds for Neurology	19.44
39 beds for Neurology	19.38
51 beds for General Surgery	19.38
21 beds for ENT	19.65
31 beds for Neurosurgery	19.75
11 beds for Ophthalmology	19.92
39 beds for Orthopedics	19.43
41 beds for Orthopedics	19.62
21 beds for Plastic Surgery	19.43
19 beds for Plastic Surgery	19.67
25 beds for Thoracic Surgery	19.91
21 beds for Urology	19.97

* Costs for allocation with $(s_j + a_j)$ beds for service j is the minimum costs of all neighboring allocations $(s_1 + a_1, s_2 + a_2, \dots, s_j + a_j, \dots, s_9 + a_9)$ which do not violate the constraints, where

$$\sum_{i \neq j} a_i = -a_j ,$$

$a_i = 0, -1, \text{ or } +1$ for $i \neq j$, and

$a_j = -1, +1$.

CHAPTER FOUR

THE SCHEDULING OF ADMISSIONS

The bed allocation procedure, presented in Chapter Three, provides a solution for distributing beds among services to achieve the minimum costs for the hospital. However, with beds optimally allocated, the hospital can still operate at a high cost due to fluctuations in the daily census if the admissions process is left to follow its own random nature. Controls on elective admissions can be used to keep the variation in daily census at the minimum level, thus helping to reduce operating costs. The census prediction model provides a tool for determining the number of elective patients to be admitted in order to attain desired census levels and smooth out variation in census. In this chapter, essential elements of the admissions process are studied and a census prediction model to forecast future bed utilization is developed as a function of these elements. Different techniques for estimating the components of the census prediction process are presented with prediction results for various hospitals studied. From these results, the characteristics of each hospital and their effects on census prediction are identified. In the last part of the chapter, the use of census prediction as means of scheduling patients is discussed for two general categories of hospitals: underbedded and overbedded.

Data used in the study were collected for four different hospitals in the Gainesville, Florida, metropolitan area. A summary of the data

collection and data description for each individual hospital is presented in Appendix C.

4.1 A General Description of the Admissions Process

Systems of referral and admission vary widely between hospitals. However, the main components of the system are patient arrivals and discharges. Patients can be classified into two basic categories: elective and emergency. Some hospitals also include urgent patients. These classifications are based on medical judgments made by the attending physician and are related to how soon the patient must be hospitalized. Normally, emergency patients must be admitted immediately and urgent patients must be admitted within 24 to 48 hours. Only elective patients, whose hospitalization can be delayed, are subject to scheduling at some future date.

The number of requests for admission per day is a random variable commonly assumed to be stationary or, more realistically, nonstationary Poisson distributed (63). The number of actual admissions per day consists of emergency and scheduled elective patients. Emergency admissions retain unpredictable characteristics, while elective admissions are usually controlled by some specified hospital policies.

Once a patient is admitted to the hospital, his length of stay is also a random variable. Physicians are often asked to predict the length of stay of admitted patients, but no one really knows exactly how long a patient will stay in the hospital until the final discharge decision by the physician. Sometimes, this decision is not made until a few hours before the patient is discharged. Therefore, little control or planning can be imposed on the discharge process and census control must be based on the admissions decision.

A hospital consists of many professional services. Each service has its own admissions demand and length of stay characteristics. In this research, admissions controls are considered for each subunit (service), the overall control of the hospital is composed of all subunits held together by constraints on total resources and by interactions (transfers) among subunits. A general system for hospital admissions is described in Figure 4.1.

It is important to define the objectives of an admissions system. Each hospital has its own objectives to achieve depending on its operating settings and situation. In general, these objectives depend on whether the hospital is overbedded or underbedded. An overbedded hospital has more beds available than the demand for beds most of the time. Hancock *et al.* (30) give general objectives for the overbedded and underbedded hospitals. For an overbedded hospital, controls of admissions are often used to minimize variation in the daily census and to schedule as many patients as possible subject to the constraints of nursing hours per patient day, admissions delay and the weekend census. For an underbedded hospital, the objectives are to maximize the census and schedule as many patients as possible subject to the constraints of cancellations, turnaways and the weekend census.

A model for predicting census based on the prediction of two random variables of the process, emergency arrivals and daily discharges, is described in the following section.

4.2 A Census Prediction Model

A census prediction model is constructed to forecast census on future dates based on the current census, the elective patients already scheduled

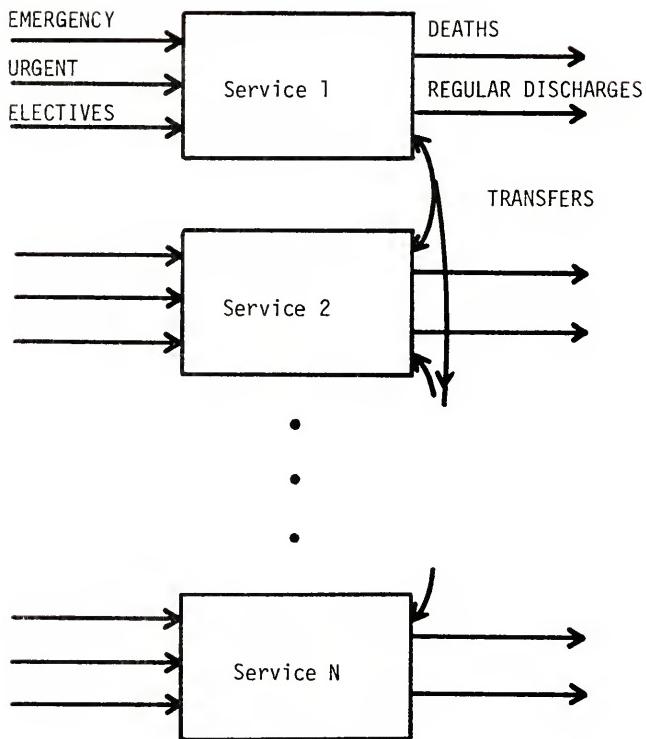


Figure 4.1 A general hospital admissions system.

for future dates, the estimated number of emergency arrivals for any future date, and the anticipated daily discharges. The approach used here is similar to that of Swain (62) and Rubenstein (59). The predicted census is found for each service and combined for the total census of the hospital. The model is designed to predict daily the expected number of emergency arrivals and the expected number of discharges based on historical data. A sketch of the components of the census prediction model and their flows in time for the hospital is presented in Figure 4.2. For the moment, it is assumed that the patient length of stay distributions have been predetermined for each service and that the expected number of emergency arrivals has been estimated by some forecasting method. Discussions of the length of stay and emergency arrivals are presented later in the chapter. The census prediction process is described analytically in the following section.

Let the census at the start of day 1 be c_0 . The census of the next day is the total of: 1) the patients currently in the hospital and not discharged for at least one or more days; 2) the scheduled admissions for the day, s_1 , and 3) the emergency arrivals for the day, e_1 . The patients currently in the hospital who remain for at least another day can be expressed by $[c_0 - d_1(c_0)]$, where $d_1(c_0)$ is the discharges for the day from the current population c_0 . The census at the start of the next day can be expressed in the following equation

$$c_1 = c_0 - d_1(c_0) + s_1 + e_1$$

Similarly, the census of successive days c_i , $i = 2, 3, \dots$, can be found:

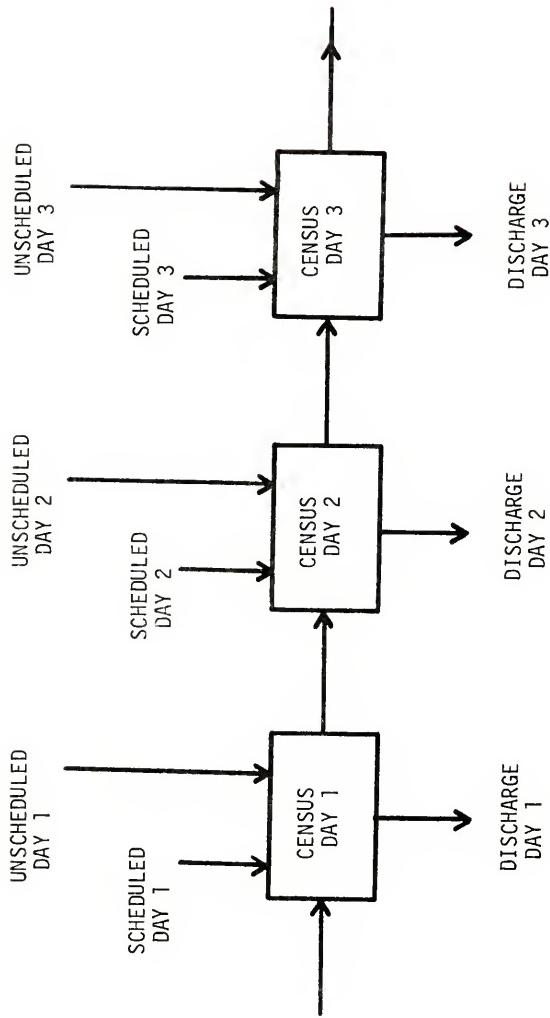


Figure 4.2 The flow of census in time.

$$\left. \begin{aligned} c_2 &= c_1 - d_2(c_1) + s_2 + e_2 \\ &\vdots \\ c_i &= c_{i-1} - d_i(c_{i-1}) + s_i + e_i \end{aligned} \right\} (4.1)$$

The recursive relationships of the above equations can be expressed as

$$\left. \begin{aligned} c_2 &= c_0 - d_1(c_0) - d_2(c_0) + s_1 - d_2(s_1) + e_1 - d_2(e_1) + s_2 + e_2 \\ &\vdots \\ c_i &= c_0 - \sum_{j=1}^i d_j(c_0) + \sum_{k=1}^{i-1} [s_k - \sum_{j=k+1}^i d_j(s_k)] + \sum_{k=1}^{i-1} [e_k - \sum_{j=k+1}^i d_j(e_k)] \\ &\quad + s_i + e_i \end{aligned} \right\} (4.2)$$

In other words, the census on day i in the future can be expressed as the patients remaining in the current population, the scheduled admissions up through day i , and the emergency admissions up through day i .

The number of emergency admissions per day is a random variable. The number of discharges depends on the length of stay distributions; therefore, it is also a random variable. As a result, in the above set of equations, (4.2), c_i is randomly distributed with a probability distribution that can be found by convolving the probabilities of each term on the right hand side of the equations. This exact calculation of the census distribution requires a convolution of probability functions which is usually a costly computation. An approximate method for obtaining the census distribution of day i in the future has been used in many studies (59,62,7). This method approximates the census distribution by a normal distribution with an evaluated mean and variance. The mean value of the census can be determined by using the best estimates of emergency arrivals and of daily discharges. Methods for

estimating emergency arrivals are discussed in Section 4.3. The method for evaluating the number of discharges is presented here.

It is recognized that a patient's stay in the hospital is a memory process, that is, the probability of additional days of stay is a function of the number of days that a patient has already spent in the hospital. In the four hospitals studied, the number of discharges has a weekly cyclic pattern. Therefore, it has been found that it is more convenient to use the probability of being discharged on a certain day to adjust for the day of the week effect than to use the probability of remaining in the hospital as in Rubenstein (59) and Swain (62). The probability of remaining exactly another j days for a patient is the conditional probability of the number of days t that the patient has already stayed:

$$P_{tj} = P[L = t+j \mid L > t]$$

where L is the length of stay, and P_{tj} is the probability of staying ($t+j$) days, or the probability of being discharged on day j . The conditional probability P_{tj} is assumed to be available for the moment, methods for evaluating P_{tj} are presented in Section 4.4.3.

Consider, for example, an arbitrary service which consists of patients who have already stayed for t days, $t=1,2,\dots,t_{\max}$. Let $n(t)$ be the number of patients of t -day stay. Assuming that the length of stay of these patients are statistically independent, the expected number of patients from this population discharged on day j is

$$E[d_j(n(t))] = n(t)P_{tj},$$

and the corresponding variance is

$$V[d_j(n(t))] = n(t)P_{tj}[1-P_{tj}].$$

The expected number of patients remaining on day i and the corresponding variance are

$$E[C_i^0] = \sum_{t=1}^{t_{\max}} [n(t) - n(t) \sum_{j=1}^i p_{tj}] , \quad (4.3)$$

$$V[C_i^0] = \sum_{t=1}^{t_{\max}} n(t) \left[\sum_{j=1}^i p_{tj} \right] \left[1 - \sum_{j=1}^i p_{tj} \right] . \quad (4.4)$$

A similar analysis can be conducted for elective patients scheduled on day k discharged on day j

$$E[d_j(s_k)] = s_k p_{0,j-k} ,$$

and the corresponding variance is

$$V[d_j(s_k)] = s_k p_{0,j-k} [1 - p_{0,j-k}] .$$

The expected number of scheduled patients on day k remaining on day i and the variance can be found as

$$E[S_i^k] = s_k - s_k \sum_{j=k+1}^i p_{0,j-k} , \quad (4.5)$$

$$V[S_i^k] = s_k \left[\sum_{j=k+1}^i p_{0,j-k} \right] \left[1 - \sum_{j=k+1}^i p_{0,j-k} \right] . \quad (4.6)$$

Emergency admissions are randomly distributed, therefore, the expected number of emergency patients on day k who are discharged on day j is determined as follows, using the results in Appendix D,

$$E[d_j(e_k)] = e_k p_{0,j-k} ,$$

and the corresponding variance is

$$V[d_j(e_k)] = e_k p_{0,j-k} [1 - p_{0,j-k}] + \sigma_k^2 [p_{0,j-k}]^2 ,$$

where e_k , σ_k^2 are the mean value and variance of the emergency arrivals, respectively.

The expected number of emergency patients on day k remaining on day i and the variance are

$$E[\varepsilon_i^k] = [e_k - e_k \sum_{j=k+1}^i p_{0,j-k}] , \quad (4.7)$$

$$V[\varepsilon_i^k] = e_k \left[\sum_{j=k+1}^i p_{0,j-k} \right] [1 - \sum_{j=k+1}^i p_{0,j-k}] + \sigma_k^2 \left[\sum_{j=k+1}^i p_{0,j-k} \right]^2 . \quad (4.8)$$

The expected number of total patients on day i is the sum of the expected numbers of patients in the current, the scheduled and the emergency populations, assuming that the length of stay of all patients are statistically independent (59,62,7). The expected value and variance of the number of patients in the hospital on day i are as follows

$$E[c_i] = E[c_i^0] + \sum_{k=1}^i E[s_i^k] + \sum_{k=1}^i E[\varepsilon_i^k] , \quad (4.9)$$

$$V[c_i] = V[c_i^0] + \sum_{k=1}^i V[s_i^k] + \sum_{k=1}^i V[\varepsilon_i^k] . \quad (4.10)$$

Assuming that the census is normally distributed, a 95% confidence interval of the prediction for day i can be found as

$$E[c_i] \pm 1.96 \sqrt{V[c_i]}$$

The confidence interval width increases as the prediction day is further into the future since the variance includes more uncertainty on the admissions and discharges. The expected value and variance of the number of patients in the hospital on day i for all services are found by the sum of the expected values and variances for individual services.

The expected census requires two predicted values: one for emergency arrivals and one for daily discharges. The following two sections will discuss methods that can be used to obtain these prediction values.

4.3 Methods for Predicting Unscheduled Admissions

In some hospitals, emergency and urgent admissions are not distinguished from unscheduled arrivals. Therefore, in this research, the unscheduled admissions are used in general to indicate all patients who were not scheduled before admission day.

The simplest method, presented in the literature (59,62,7), for estimating the number of unscheduled arrivals for any date in the future is to use the mean value of arrivals from historical data. It has been observed from all four hospitals in this study that the unscheduled data has a weekly cyclic pattern, therefore the mean value of arrivals by day of the week can be used to improve the estimate. One drawback of the mean value of the admissions method is that the estimates are static and cannot account for any dynamic change in time which usually occurs.

The problem can be overcome by using a seasonal time series model, which is able to pick up any cyclic pattern of the data and any change in admissions over time. The autocorrelation function is first calculated from the unscheduled data to recognize any cyclic pattern and the corresponding period. The period and estimated periodic function of the data are input to a forecasting model for predicting unscheduled arrivals on any future data based on current knowledge of the arrival process.

Let $F(1)$, $F(2)$, ..., $F(p)$ be the multiplicative seasonal factors of period p . The time series can be represented by Winters' seasonality model (72):

$$X_t = A_t * F(t) + \epsilon_t$$

where X_t is the number of unscheduled arrivals on day t , A_t is the parameter of the model usually called the permanent component, and ε_t is the random error component. Let \hat{A}_t and $\hat{F}(t)$ indicate the estimates of A_t and $F(t)$. Assuming that \hat{A}_{t-1} , $\hat{F}(t-1)$, ..., $\hat{F}(t-p)$ are known, then the estimates of A_t and $F(t)$ can be found as

$$\hat{A}_t = \alpha[X_t/\hat{F}(t-p)] + [1-\alpha]\hat{A}_{t-1} \quad (4.11)$$

$$\hat{F}(t) = \beta[X_t/\hat{A}_t] + [1-\beta]\hat{F}(t-p) \quad (4.12)$$

where α and β are smoothing factors having values in the range of $(0, 0.5)$. The two equations (4.11) and (4.12) represent smoothing of an estimate based on information prior to the current period. They were developed heuristically (72), rather than formally through the use of a criterion such as least squares.

Estimates of the unscheduled admissions on future dates \hat{X}_{t+i} , $i=1, 2, \dots$, can be computed as

$$\hat{X}_{t+i} = \hat{A}_t * \hat{F}(t+i-p).$$

The values of \hat{A}_t and $\hat{F}(t)$ are adjusted along with updated data on X_t as t varies. Therefore, the forecasting model can absorb any dynamic change in the process. In other words, the time series model predicts the unscheduled arrivals using the adaptive mean values by day of the week for the model with a period of 7. An example of the forecasted arrivals is presented in Figure 4.3, together with the actual arrivals. Data used in this example were collected from the admissions process at the Gainesville Veterans Administration Hospital from April to August of 1976. The mean error was -0.459, standard deviation of error 5.6.

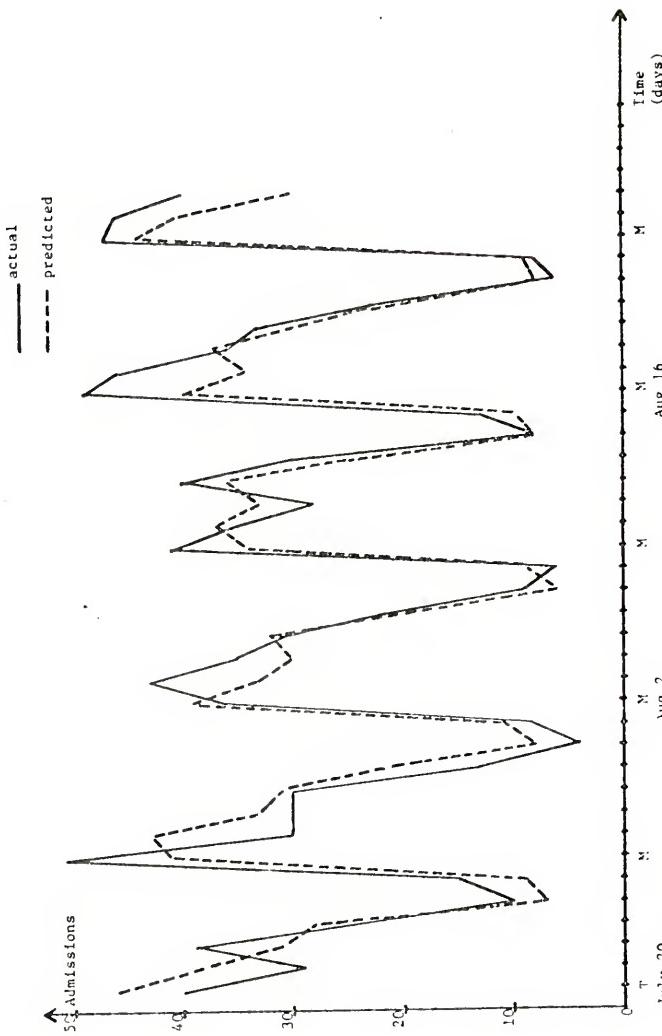


Figure 4.3. An example of admissions prediction using Winters' time series model for Gainesville Veterans Administration Hospital.

In addition to the methods presented previously, another method has been found to be useful for hospitals with a small number of unscheduled arrivals. The scheduled arrivals often account for a great part of the admissions. The number of daily unscheduled arrivals is usually small and has a value of zero for a large percentage of time. The use of the mean value for prediction, therefore, can cause large errors most of the time. It has been observed that there usually exists a significant negative correlation between the numbers of unscheduled and scheduled arrivals. When the number of scheduled admissions is large, the number of unscheduled admissions for the day tends to be small. When the number of scheduled reservations is small, the number of unscheduled admissions for the day tends to be high due to availability of beds in the hospital. If the "no shows," patients that do not keep their scheduled reservation, are considered as negative unscheduled admissions, the negative correlation between the unscheduled and scheduled admissions can be readily explained. The number of no shows is expected to increase with the number of scheduled reservations; thus, for a large number of scheduled reservations, the unscheduled admissions are expected to be more negative. The unscheduled and scheduled admissions for Pediatric Medicine, E.N.T., and Surgery services are plotted in Figure 4.4 together with the least square lines. Data on admissions were collected for 50 days. The correlation between unscheduled and scheduled were found to be negative with correlation coefficients -.23, -.79, and -.05, for Pediatric Medicine, E.N.T., and Surgery services, respectively. The correlation method improves the estimates of unscheduled admissions over a short period of prediction. However, for a longer period of prediction the mean value estimates tend to average out the error over the week better than the correlation estimates. The correlation estimates can be improved by

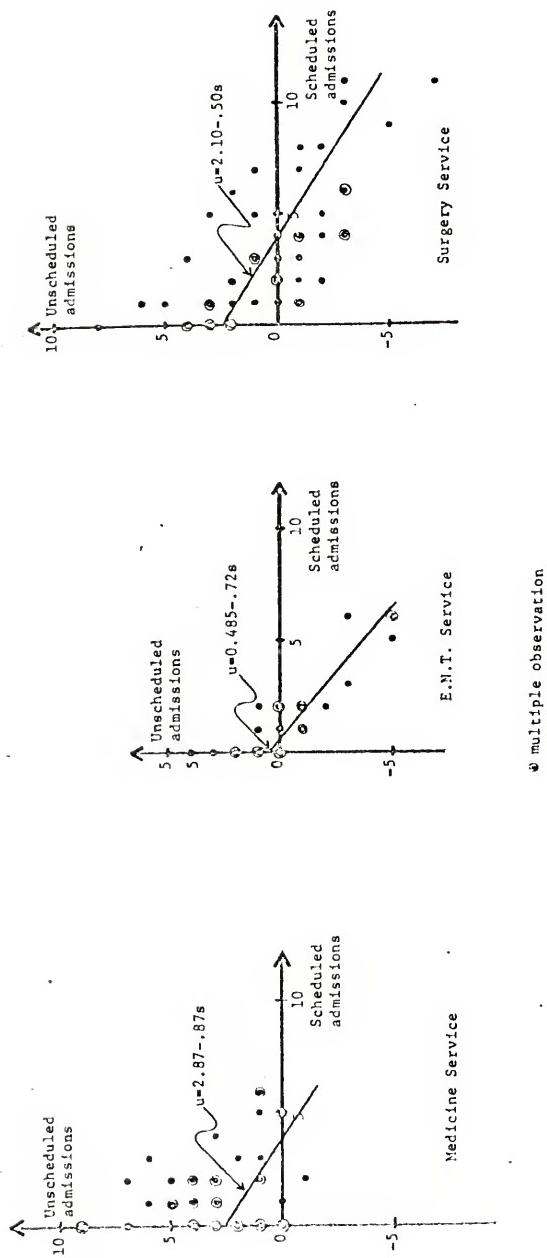


Figure 4.4. Scatter diagrams for unscheduled and scheduled admissions and the least square lines, Hands Teaching Hospital Pediatrics unit.

separating weekdays and weekend data so that the weekend effect can be smoothed out.

4.4 Methods for Predicting Patients' Length of Stay

Another component which must be predicted in the model is the length of stay of patients. This prediction is based on historical length of stay data. In the first section analysis of various parameters is presented to identify the ones which have an influence on the length of stay. The characteristics of the patient length of stay are discussed in the succeeding section. The method for predicting the length of stay is then based on the results of these analyses.

4.4.1 Parameters that Influence the Length of Stay

The patient population is highly nonhomogeneous. There are no two individual patients who have identical conditions leading to a same length of stay. The purpose of this section is, therefore, to separate the patient population into more effective groupings to predict the most accurate length of stay. Statistical tests are used to analyze data to identify the parameters that have significant effects on the length of stay. These parameters can then be considered as characteristics of the patient grouping and used as variables for the length of stay prediction.

Hospitals commonly group patients into service, therefore in this study, the patient's service is chosen to be the first variable to test for significant influence on the length of stay. The F ratio was used to test the null hypothesis, $H_0: \beta_1 = 0$, where β_0 and β_1 are parameters of the linear model

$$\text{LOS} = \beta_0 + \beta_1 \text{ SERVICE} .$$

The Statistical Analysis System package was used to obtain the values of F ratio and significant level, α . Results of the test performed on the length of stay data for the Gainesville Veterans Administration Hospital from July 1974 to June 1975 are listed in Table 4.1. It can be seen that the null hypothesis is rejected with a high level of confidence for all admissions and admissions on all days except for Wednesday. In other words, the mean length of stay is a function of the patient's service. Thus the patient's service has significant influence on the length of stay.

Some of the demographic variables, which have been recognized as having significant influence on length of stay are age and sex (14,2,54). Other variables include attending physicians, diagnoses, and the day of the week that the patients are admitted. Since the patient's service has been shown to have significant influence on the length of stay, statistical tests were performed for these variables within the sample population for these variables for individual services. Multiple regressions on these variables were performed; however, due to a large number of missing values, the tests were discarded. For each variable cited above, the F ratio was again used to test the null hypothesis, $H_0: \beta_1=0$, where β_0 and β_1 are parameter of the linear model

$$\text{LOS} = \beta_0 + \beta_1 \text{ VARIABLE} .$$

Results of the tests performed on the length of stay data for the Shands Teaching Hospital Pediatrics unit from October 1975 to May 1976 are listed in Table 4.2. The F ratio has value close to 1 for the tests of effects of attending physicians on the length of stay, and the significance levels for these tests are large; thus the null hypothesis cannot be rejected with a high level of confidence. For the effects of multiple and single

Table 4.1

Results of Testing Hypothesis $H_0: \beta_1 = 0$ for Service,
Gainesville Veterans Administration Hospital.

Sample description	Sample size	F ratio	Significance level
All admissions	10056	48.350	.0001
Monday admissions	2241	19.143	.0001
Tuesday admissions	2095	7.238	.0072
Wednesday admissions	1767	.242	.6227
Thursday admissions	1712	17.426	.0001
Friday admissions	1205	7.578	.0060
Saturday admissions	435	3.792	.0521
Sunday admissions	601	2.818	.0937

Table 4.2

Results of Testing Hypothesis $H_0: \beta_1 = 0$

Variables	F Ratio	Significance level	Sample size	Sample description
Day of the week	.347	.5561	685	Medicine
Day of the week	.776	.3795	215	Surgery
Day of the week	2.044	.1531	1098	Pediatrics
Day of the week	2.720	.0991	10056	G.V.A.H., all services
<hr/>				
Attending physician	1.904	.1679	1098	Pediatrics
Attending physician	.158	.6910	220	Ped., Monday
Attending physician	.735	.3924	179	Ped., Tuesday
Attending physician	.379	.5390	188	Ped., Wed.
Attending physician	.479	.4899	152	Ped., Thursday
Attending physician	.941	.3339	123	Ped., Friday
Attending physician	1.927	.1685	91	Ped., Sat.
Attending physician	.621	.5176	145	Ped., Sunday
<hr/>				
Diagnoses (m/s)	13.153	.0004	240	Ped., Monday
Diagnoses (m/s)	8.001	.0051	225	Ped., Tuesday
Diagnoses (m/s)	6.114	.0143	198	Ped., Wed.
Diagnoses (m/s)	1.517	.2196	185	Ped., Thursday
Diagnoses (m/s)	3.126	.0794	133	Ped., Friday
Diagnoses (m/s)	2.675	.1054	91	Ped., Sat.
Diagnoses (m/s)	3.115	.0795	163	Ped., Sunday
Diagnoses (m/s)	39.42	.0001	1235	Ped., all admissions
<hr/>				
Diagnoses (m/s)	10.453	.0015	141	Ped. Med, Monday
Diagnoses (m/s)	7.027	.0091	126	Ped. Med., Tuesday
Diagnoses (m/s)	9.317	.0028	112	Ped. Med., Wed.
Diagnoses (m/s)	.512	.4722	99	Ped. Med., Thursday
Diagnoses (m/s)	1.854	.1772	80	Ped. Med., Friday
Diagnoses (m/s)	5.375	.0245	56	Ped. Med., Sat.
Diagnoses (m/s)	.733	.3945	82	Ped. Med., Sunday

diagnoses on the length of stay, the F ratio has values much larger than 1, the significance levels are less than .10 for most of admission days. Therefore, the null hypothesis is rejected, that is, the diagnoses--multiple and single--have significant influence on the length of stay. The single and multiple diagnoses can be used as one of the parameters that characterize the length of stay. However, the diagnostic parameter is available at discharge time only; the admitting diagnoses are usually insufficiently correlated with the discharge diagnoses to be useful as a length of stay predictor.

It is recognized that the day of the week a patient is admitted has a significant influence on the length of stay. If the expected length of stay of a patient is, for example, five days, and the expected discharge day falls on a Sunday, the patient is likely to be discharged one day earlier or later due to the effect of weekends. However, in the results of the F ratio test shown in Table 4.2, the day of the week has no significant effect on the length of stay for patients in Medicine, Surgery, and Pediatrics services of Shands Teaching Hospital. For all patients in the Gainesville Veterans Administration Hospital the day of the week effect is significant at level $\alpha = .10$. These results can be explained by the fact that the day of the week effect exists but may not be detected statistically because the weekend effect may prolong some length of stay but also shorten others. Thus, the results are averaged out and no effect can be shown. The day of the week of the admission is a potential predictor for the length of stay; therefore, it is recommended that the length of stay by day of the week be used in the prediction model. The mean values of the length of stay by the day of the week of admission are plotted in Figure 4.5 for Medicine, Surgery and Orthopedics services of

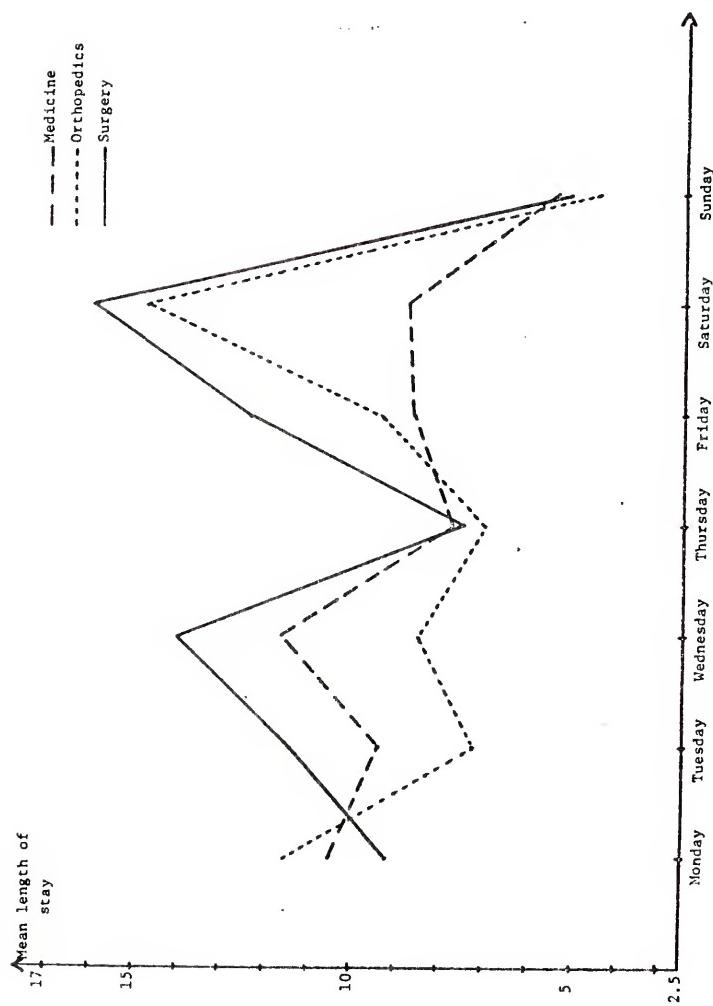


Figure 4.5. Mean values of length of stay by day of the week, Gainesville Veterans Administration Hospital.

the Gainesville Veterans Administration Hospital. It can be seen that there is a significant difference between the mean lengths of stay by the day of the week of admission.

In summary, the length of stay of patients is found to be characterized best by professional services and by the day of the week of admission.

4.4.2 Length of Stay Distributions

Data on the length of stay of patients are grouped into services and day of admission. Each group is tested separately for the compatibility with different theoretical distribution functions. Attempts to find theoretical distributions that fit the length of stay data are made since theoretical distributions are usually easier to work with and do not require as much computer memory space as empirical distributions. Moreover, analytic solutions may be developed with theoretical distributions. The chi-square test is used to measure the compatibility of the empirical and theoretical frequencies of some continuous distributions such as gamma, Weibull, normal, lognormal, and inverse Gaussian distributions, as well as some selected discrete distributions such as negative binomial and geometric distributions. The length of stay of patients is commonly taken to be the number of days the patients stayed in the hospital. Thus it is a discrete function of time. The continuous distributions are used as approximation of the discrete length of stay function. The theoretical distributions have been chosen either because of the similarity in the shape of the probability density (or mass) functions, or due to their use in the literature. The theoretical distributions are briefly described on the following pages. Actual cumulative distribution functions were generated according to Fishman (25).

The gamma distribution: The mean and variance of the observed data are used to find the two parameters, λ and α , of the gamma distribution. The probability density function can be obtained accordingly

$$\alpha = \frac{\mu^2}{\sigma^2}$$

$$\lambda = \frac{\mu}{\sigma^2}$$

$$F_T(t) = \frac{\lambda^\alpha t^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda t}$$

where μ is the mean and σ^2 the variance of the population.

The Weibull distribution: The Weibull distribution is often used to represent the failure distribution. The length of stay can be considered as the time to failure where failure is the event that the patient is discharged. The cumulative probability distribution is

$$F_T(t) = 1 - e^{-(\lambda t)^\alpha}$$

where the parameter α can be found as the slope of the line

$$Y = \alpha x + b$$

where $Y = \ln\left(\frac{1}{1 - F_T(t)}\right)$,

$$x = \ln t, \text{ and}$$

$$b = \alpha \ln \lambda .$$

The normal distribution: The probability distribution function of the normal distribution with mean μ and variance σ^2 can be obtained by a transformation of the standard normal

$$F_T(t) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

where $\Phi(\cdot)$ is the standard normal distribution function with mean 0 and variance 1.

The lognormal distribution: A random variable T is distributed as a lognormal distribution if x is a normal random variate and

$$T = e^X .$$

The probability distribution function of a lognormal random variable is

$$F_T(t) = \Phi\left(\frac{\ln t - M}{\sqrt{V}}\right)$$

where $M = \ln \mu - \frac{v}{2}$, and

$$v = \ln(\sigma^2 + \mu^2) - 2 \ln \mu.$$

The inverse Gaussian distribution: The probability density function of the inverse Gaussian can be found as

$$f_T(t) = \left(\frac{\lambda}{2\pi t^3}\right)^{1/2} \exp\left(-\frac{\lambda(t-\mu)^2}{2\mu^2 t}\right) \quad t > 0$$

where μ is the population mean, and λ is a parameter that measures relative precision.

The negative binomial distribution: The probability mass function of the negative binomial distribution is

$$P(t) = \binom{r+t-1}{t} p^r q^t \quad t = 0, 1, 2, \dots,$$

where the parameters r and p are defined in terms of the mean and variance of the population as

$$p = \frac{\mu}{\sigma^2}$$

$$r = \frac{\mu^2}{\sigma^2 - \mu} .$$

The geometric distribution: The probability mass function of the geometric distribution is

$$P(t) = pq^{t-1} \quad t = 1, 2, \dots$$

$$\text{where } p = \frac{1}{\mu} .$$

The chi-square test for goodness of fit was performed on the length of stay data for all of the above theoretical distribution. Results on the length of stay for the Pediatric Surgery and Medicine services of the Shands Teaching Hospital are presented in Table 4.3. From these results, the χ^2 values of the testing goodness of fit to all theoretical distributions are much larger than the χ^2 values at significant level $\alpha = .10$. The increasing large values of χ^2 may be thought of as corresponding to increasingly poor compatibility of observed and expected theoretical frequencies. The '*' in the table indicates the best χ^2 value for each set of observed data. The inverse Gaussian, Weibull, negative binomial, and geometric distributions are shown to have smallest χ^2 values for some service; however, their χ^2 values are at unacceptable significance level, i.e., $\alpha << .10$. The test performed on Pediatric Surgery service for Friday admissions gave large values of χ^2 . This can be explained by the fact that 48 observed data are spread through 40 different values which caused a small number of observations at each length of stay value. Since the χ^2 discrete frequency function is an approximation of the continuous χ^2 distribution function, the larger the number of observations in each

Table 4.3
Results of the Chi-Square for Testing Goodness of Fit

Sample description	Sample Size	Normal			Log Normal			Inverse Gaussian			Weibull			Negative Binomial			Geometric			Gamma		
		χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	$\chi^2_{.90}$	χ^2	d.f.	
Tuesday	114	147.26	17	10.09	157.73	17	10.09	99.21	17	10.09	49.06*	17	10.09	62.36	17	10.09	51.91	18	10.86	53.72	17	10.09
Wednesday	101	162.81	14	7.79	68.64	14	7.79	61.79	14	7.79	58.35	14	7.79	67.41	14	7.79	53.15*	15	8.55	60.18	14	7.79
Thursday	90	86.74	18	10.09	209.60	18	10.86	369.46	18	10.86	28.87*	18	10.86	39.65	18	10.86	31.41	19	11.65	38.23	18	10.86
Friday	72	67.73	22	14.04	164.40	22	14.04	588.24	22	14.04	29.18	22	14.04	30.71	22	14.04	28.25*	23	14.85	31.17	22	14.04
Saturday	51	42.73	18	10.86	246.60	18	10.86	380.90	18	10.86	28.05*	18	10.86	33.01	18	10.86	37.52	19	11.65	30.25	18	10.86
Sunday	74	132.60	10	4.87	42.58	10	4.87	37.71	10	4.87	66.08	10	4.87	68.01	10	4.87	54.81	11	5.58	56.13	10	4.87
Pediatric Medical	124	71.36	9	4.17	120.52	9	4.17	169.70	9	4.17	46.93	9	4.17	39.99*	9	4.17	45.36	10	4.87	47.20	9	4.17
<hr/>																						
Tuesday	74	56.29	14	7.79	154.48	14	7.79	554.53	14	7.79	18.27*	14	7.79	26.27	14	7.79	22.26	15	8.55	19.29	14	7.79
Wednesday	41	88.87	22	14.04	73.76	22	14.04	122.86	22	14.04	29.59*	22	14.04	61.23	22	14.04	33.05	23	14.85	35.86	22	14.04
Thursday	58	63.31	11	5.58	69.69	11	5.58	94.06	11	5.58	25.86*	11	5.58	33.64	11	5.58	28.26	12	6.30	29.91	11	5.58
Friday	48	358.49	37	18.56	409.16	37	18.56	567.15	37	18.56	217.91	37	18.56	172.36	37	18.56	270.26	28	19.25	250.23	37	18.56
Saturday	27	34.97	18	10.86	37.78	18	10.86	179.14	18	10.86	44.54	18	10.86	20.39*	18	10.86	23.29	19	11.65	31.74	18	10.86
Sunday	67	52.13	10	4.87	56.49	10	4.87	376.72	10	4.87	20.79*	10	4.87	25.84	10	4.87	23.91	11	5.58	24.62	10	4.87
Monday	78	79.57	11	5.58	68.79	11	5.58	43.79	11	5.58	33.80*	11	5.58	41.57	11	5.58	38.07	12	6.30	37.29	11	5.58

length of stay value the more satisfactory the test for goodness of fit.

The inability to identify the length of stay by a theoretical distribution can be explained by the nonhomogeneity of patients which results in a mix of distributions for the length of stay data. While the empirical distribution may impede the development of any analytic solution, it gives generality to the model. In most hospitals studied, the number of services does not exceed 50, thus the computer memory space required is not considerable for the empirical length of stay distribution.

4.4.3 Mean Residual Length of Stay

In the two previous sections, the length of stay is found to be a special distribution characterized by the hospital services and by the day of admission. In this section, the mean residual length of stay provides a better explanation for the length of stay distribution and a method to further characterize the length of stay.

The mean residual length of stay is the expected additional length of stay of patients who have already stayed t days, i.e.,

$$E[L - t \mid L > t], \quad t \geq 0$$

where L is the length of stay random variable. The estimated mean residual length of stay can be found for each service as follows (51)

$$\hat{E}[L - t \mid L > t] = \frac{\sum_{k=t+1}^{\infty} (k-t)n_k}{\sum_{k=t+1}^{\infty} n_k}$$

where n_k is the number of patients in the service staying k days which can be obtained from past length of stay data.

Figure 4.6 shows the mean residual life functions for the gamma, lognormal and Weibull distribution (51) with mean value $\mu=1$ and coefficient of variation $\eta = \sqrt{5}$. The coefficient of variation η of L is defined as

$$\eta = \sqrt{\frac{E[L^2]}{\mu^2} - 1}$$

For $\eta < 1$, the gamma and Weibull mean residual life functions are monotonically decreasing. For $\eta = 1$, the gamma and Weibull distributions become the exponential function. For $\eta > 1$, the Weibull mean residual life function increases without bound and the gamma mean residual life function is monotonically increasing and tends to a nonzero constant as time $t \rightarrow \infty$. The lognormal mean residual life function is always decreasing at first and then increasing. A typical mean residual length of stay function is plotted in Figure 4.7 for Surgery service at Shands Teaching Hospital, based on a sample of size 1365 admissions. The mean residual length of stay function is increasing at first, then levels off at a nonzero constant. Therefore, the length of stay distribution has an exponential tail. Figures 4.8 and 4.9 show the mean residual length of stay functions for the Pediatric Medicine service for Thursday admissions and the Pediatric Surgery service for Monday admissions. These two mean residual length of stay functions stay approximately constant from the beginning, thus the length of stay distribution is approximately geometric. Figure 4.10 shows the mean residual length of stay function for the Pediatric Medicine service for Monday admissions. The mean residual length of stay is increasing at first, then starts to level off at day 9.

It can be seen from Figure 4.7 that the length of stay of patients is a memory process for a short length of stay. At some point T in time,

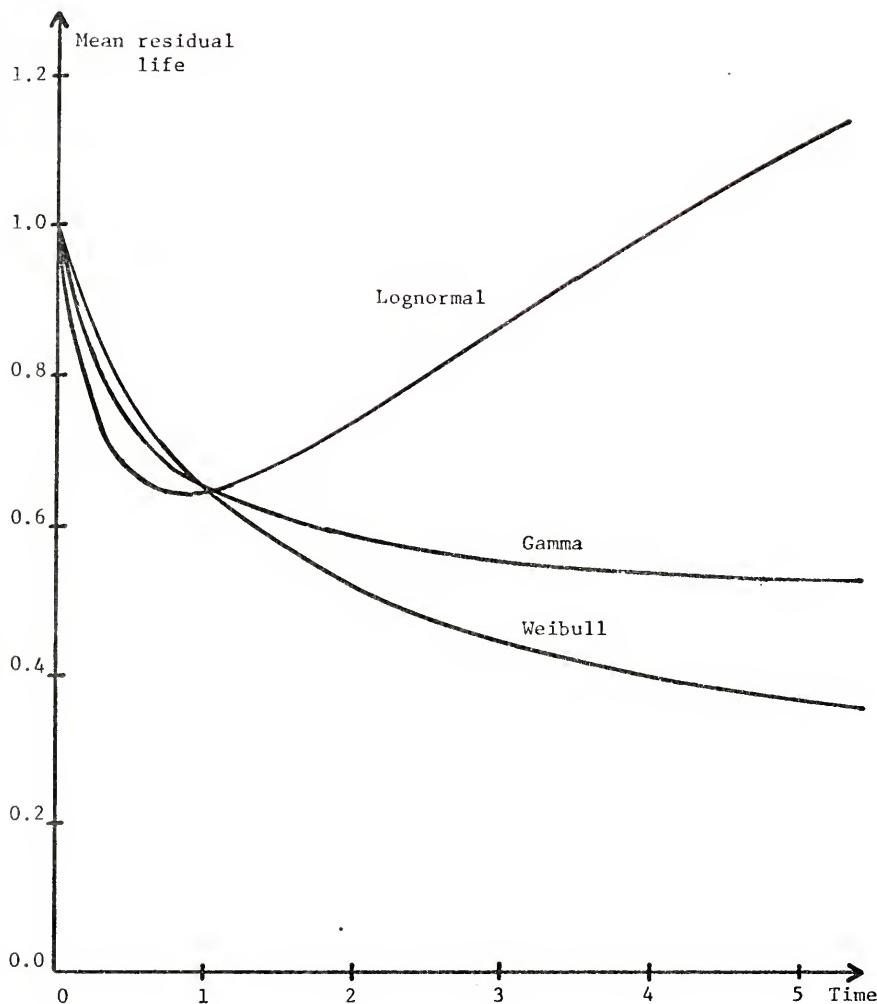


Figure 4.6. Mean residual life functions for the gamma, lognormal, and Weibull distributions, with mean 1 and coefficient of variation $\sqrt{0.5}$.

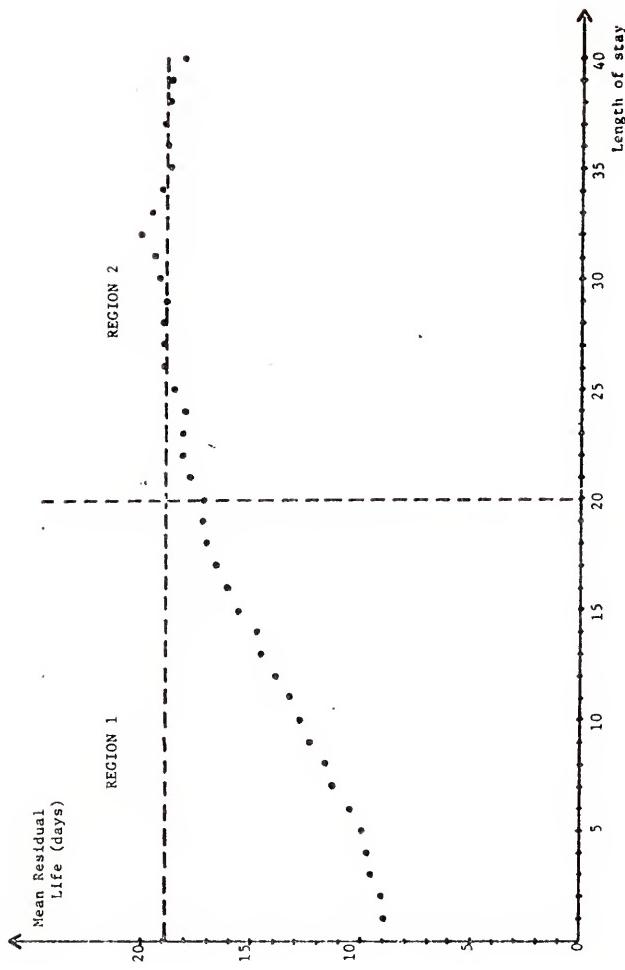


Figure 4.7. The mean residual length of stay function for Surgery service,
Shands Teaching Hospital.

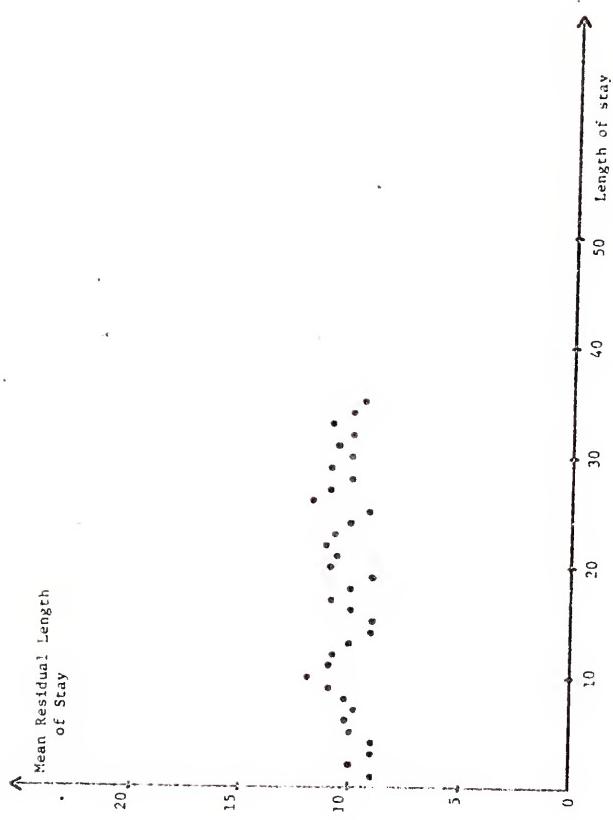


Figure 4.8 Mean residual length of stay function, Shands Teaching Hospital, Pediatric Medicine service, Thursday admissions.

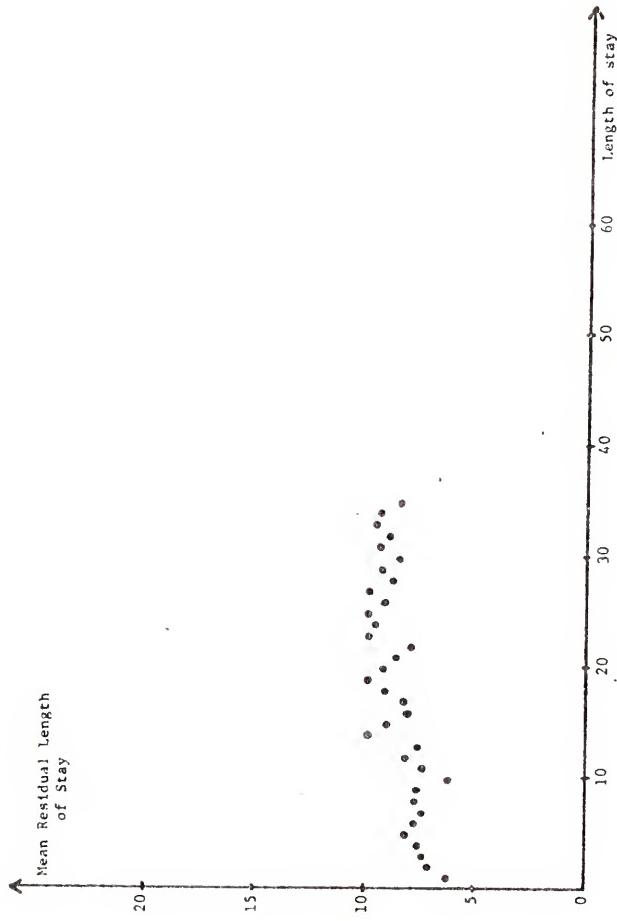


Figure 4.9. Mean residual length of stay function, Shands Teaching Hospital, Pediatric Surgery service, Monday admissions.

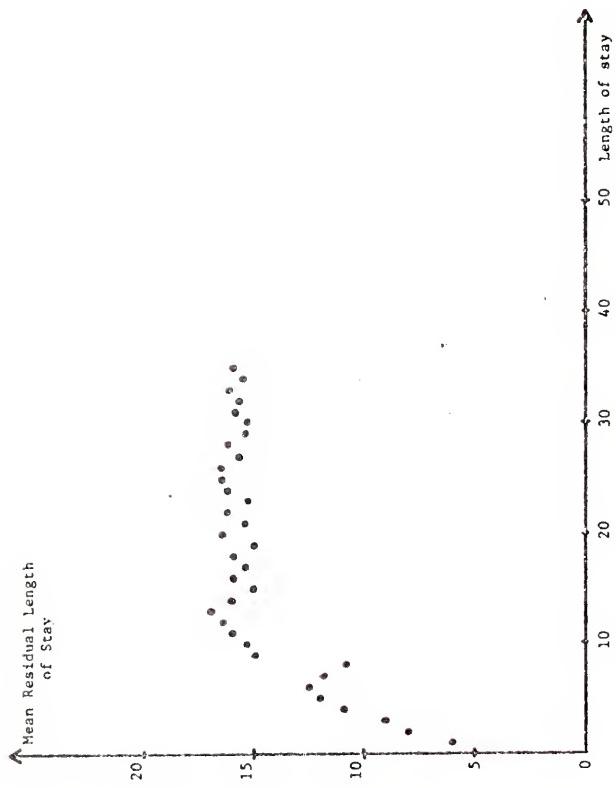


Figure 4.10. Mean residual length of stay function, Shands Teaching Hospital, Pediatric Medicine service, Monday admissions.

the process becomes memoryless and thereby the length of stay distribution becomes geometric. This point T can be determined by testing the variation of the mean residual length of stay function with time. If the mean residual length of stay values within an interval ϵ for a certain number of consecutive days τ , the mean residual length of stay function can be considered as leveling off and T can be taken to be the starting point of that period of τ days. Values of ϵ and τ are chosen arbitrarily and depend on the length of stay data of each hospital. In this study, ϵ was chosen to be 3 days and τ 10 days for Shands Teaching Hospital. The mean residual length of stay function becomes smoother as the number of observations on length of stay increases. Thus, ϵ can be chosen at a smaller value for a large sample size. The point in time T where the switch in the process occurs can be used to separate the distribution into two regions as shown in Figure 4.7. Region 1 is for the length of stay distribution of short stay patients and Region 2 is for the length of stay distribution of long stay patients. If the patient has stayed less than T days, the probability that he will stay j more days is based on the length of stay distribution of short stay patients. The probability of discharge is conditioned on the number of days that the patient has already spent in the hospital and can be expressed as follows

$$P_{tj} = P[L = t + j \mid L > t] = \frac{P[L = t + j]}{P[L > t]} \quad \text{for } t < T \quad (4.13)$$

If the patient has already stayed more than T days, then his length of stay distribution is that for long stay patients. Since the long stay patient distribution is a memoryless length of stay process, the conditional probability of stay is geometrically distributed. The discharge probability, i.e., the probability of success for the geometric distribution

can be given as follows

$$P_{tj} = P[L = t + j \mid L > t] = p(1 - p)^{j-1} \quad t \geq T \quad (4.14)$$

where p = probability of leaving the system

$$= \frac{1}{MRL}, \text{ and}$$

MRL = the mean residual length of stay for long stay patients in Region 2.

In summary, patients are classified by services and by day of admission to predict their length of stay. The number of days that the patient has already spent in the hospital is used to determine the discharge probability of the patient by the mean residual length of stay method. The discharge probability of the patient on day j can be evaluated by Equations (4.13) or (4.14) depending on the region which contains the patient.

4.5 Prediction of Future Census

The census can be projected from daily admissions and discharges as presented in Section 4.2. Tests of the census prediction model were conducted at the Shands Teaching Hospital Pediatrics unit and at the Gainesville Veterans Administration Hospital. Results of these predictions using different methods are presented in the following sections. The characteristics of each hospital and their effects on the census prediction are also discussed.

4.5.1 Results for the Shands Teaching Hospital

The Pediatrics unit at the Shands Teaching Hospital was chosen for the census prediction test because the unit is subdivided into several professional services and operates as a hospital within a hospital.

There are about 92 beds assigned to more than 12 services of the Pediatrics unit. The length of stay data were collected by day of the week of admission for each service using the condensed billing data of the hospital. At the time of the test, six-months' worth of data were available. Since the sample size was too small for some services, it was best to combine those with common length of stay distributions into groups. The Duncan Multiple Range test was used to identify services that have similar length of stay. Services in the Pediatrics unit fall into five distinct groups. Medicine, Psychiatry, and Surgery 4 have significantly different lengths of stay, thus forming three individual groups. Eye, E.N.T., and Oral Surgery have common lengths of stay and make up another group. The last group includes all other surgeries such as Neurosurgery, Orthopedics, Neurology, Surgeries 1, 2, 3, and 5. This grouping of services was also used in collecting the unscheduled admission data. The number of unscheduled admissions for the Pediatrics unit was found to be very small and no weekly pattern could be observed. Therefore, the mean number of unscheduled admissions was used for all days of the week.

The prediction process was carried out for seven weeks during the period from May 27 to July 15. The errors of prediction for the whole Pediatrics unit are plotted in Figures 4.11 and 4.12 for one-day and seven-day predictions, respectively. This test focuses on the total unit, rather than individual services since there is no specific bed assignment for each service. The frequencies of the prediction errors are also presented in Figures 4.13 and 4.14 for one-day and seven-day predictions, respectively. The empirical census prediction errors and the corresponding percentage of prediction whose error does not exceed the bounds on the error interval for

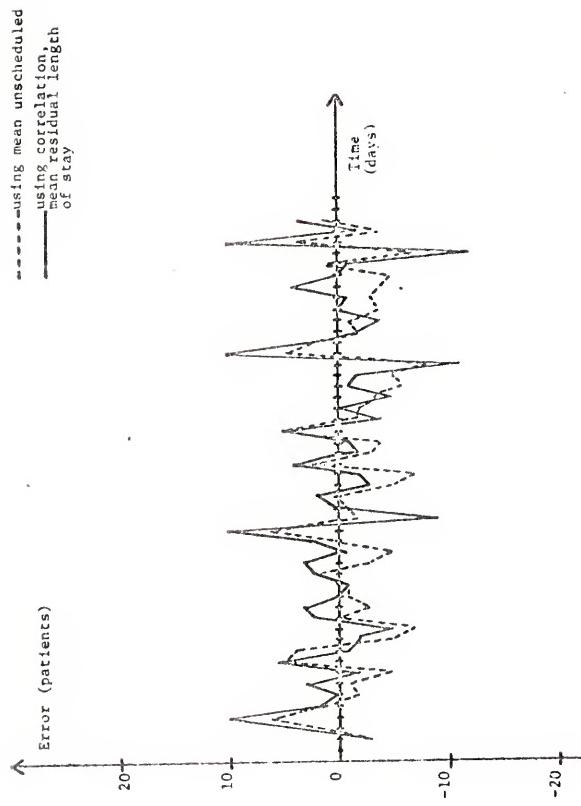


Figure 4.11. One-day census prediction errors, Shands Teaching Hospital Pediatrics unit. (Error = predicted-actual)

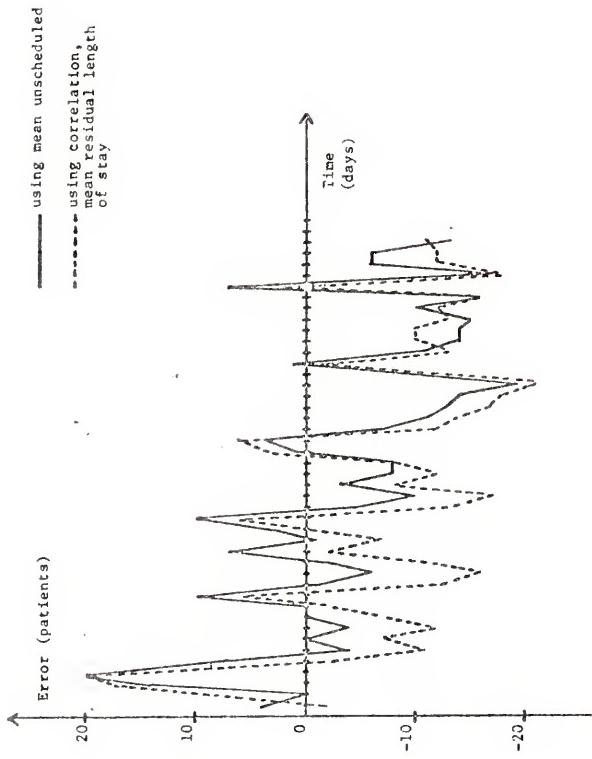


Figure 4.12. Seven-day census prediction errors, Shands Teaching Hospital Pediatrics Unit. (Error = predicted-actual.)

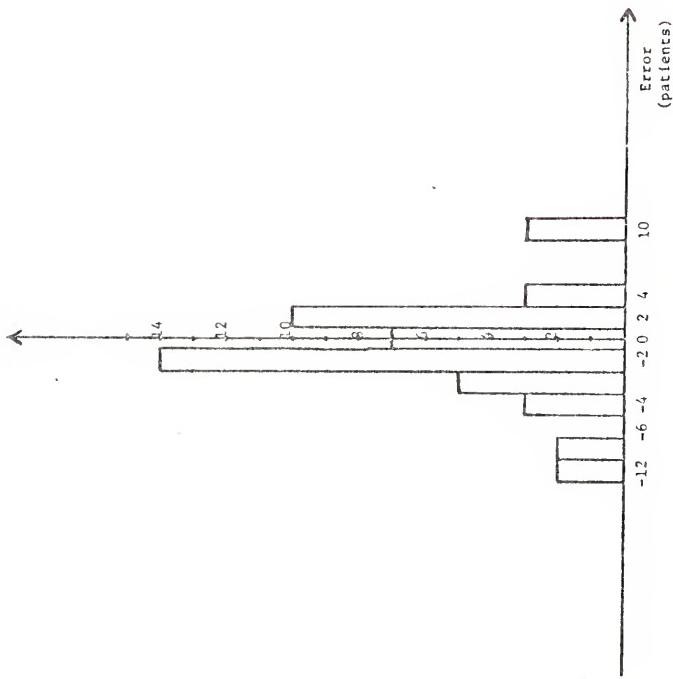


Figure 4.13. Frequency of one-day prediction errors, Shands Teaching Hospital Pediatrics unit.

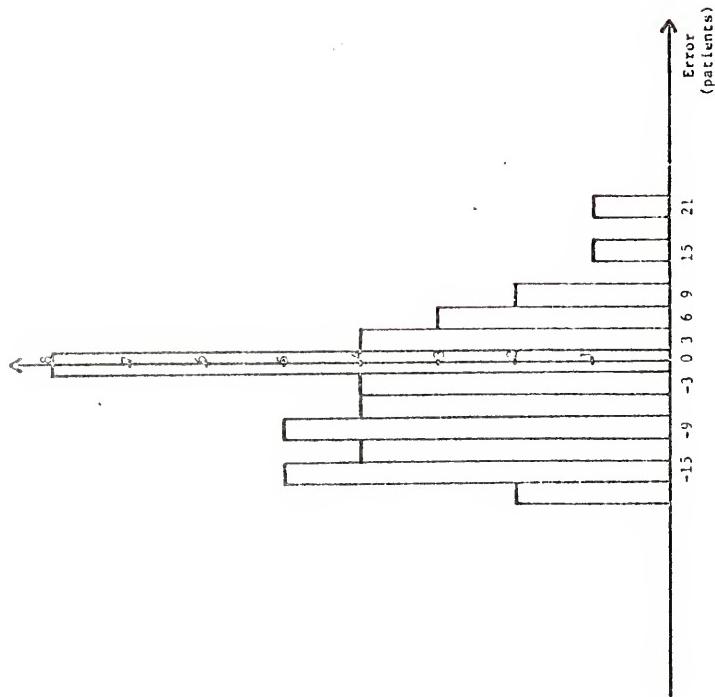


Figure 4.14. Frequency of seven-day prediction errors, Shands Teaching Hospital Pediatrics unit.

Shands Teaching Hospital Pediatrics unit are listed in Table 4.4 for a one-day census prediction. For the one-day prediction, the average error is .497 patient and the standard deviation for the sample is 4.969. An over-prediction of approximately one-half patient on the average can result from either the unscheduled admission or the discharge prediction. The errors on the unscheduled admissions and discharge prediction are plotted in Figures 4.15 and 4.16, respectively. It can be seen that unscheduled admissions are over-predicted with an average of 2.26 which accounts for the error in the census prediction. For the Shands Teaching Hospital, where admissions are based on referral, the number of unscheduled admissions is very low a large percentage of the time. Use of the average unscheduled admissions, therefore, produces an overestimate most of the time.

Use of another method for predicting the unscheduled admissions seems to be justified. A method using the correlation between the unscheduled and scheduled admissions presented in Section 4.3 is therefore applied to the prediction model. The data on unscheduled and scheduled admissions were available for a period of two months. The "no shows" were considered as negative unscheduled admissions. From these data, least square lines were found as shown in Figure 4.4. The number of admissions scheduled for day 1 to day 15 into the future were recorded by the secretary of Pediatrics unit. The unscheduled admissions expected for day 1 to day 15 into the future were found by taking the unscheduled value of the intersection of given scheduled admissions and the least square line. The errors of the unscheduled admission prediction are compared to ones using the average values in Figure 4.15. The errors of census prediction are also plotted against those of the previous prediction method in Figures 4.11 and 4.12 for one-day and seven-day predictions, respectively. The average error for

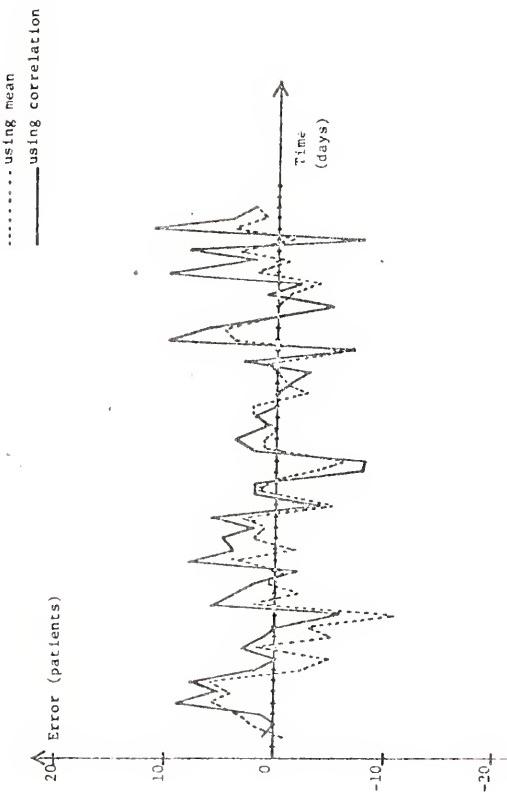


Figure 4.15. Prediction errors for admissions, Shands Teaching Hospital Pediatrics unit. (Error = predicted-actual.)

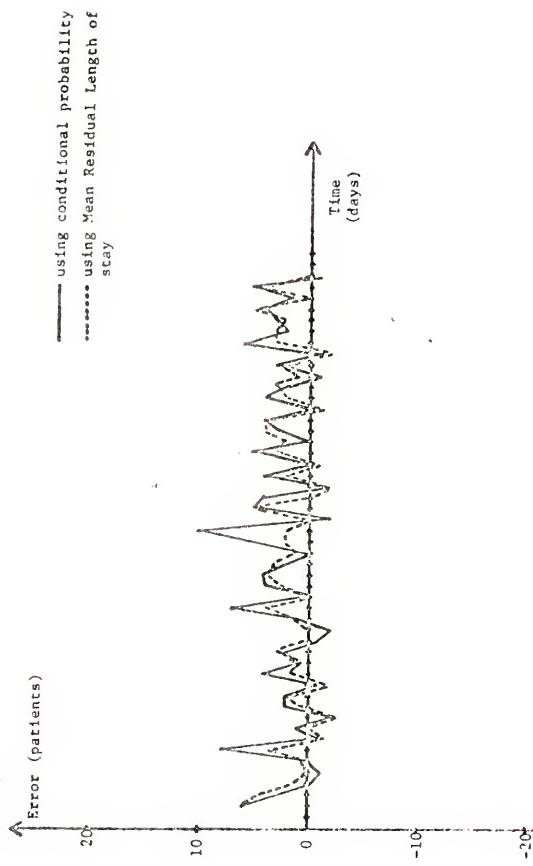


Figure 4.16. Prediction errors for discharges, Shands Teaching Hospital Pediatrics unit. (Error = predicted-actual.)

Table 4.4

The Empirical Prediction Errors for
Shands Teaching Hospital Pediatrics Unit

Empirical prediction errors (patients)	Percent of prediction whose error is within the intervals
(-2, +2)	55.10
(-5, +5)	85.72
(-10, +10)	95.92

one-day census prediction is -1.018 patients with a standard deviation of 4.347. It should be noted that while the average error is greater in absolute value, the prediction error has a smaller standard deviation which is the more desirable statistic. In comparing the two seven-day prediction errors, the correlation method gives a higher under-prediction. An under-prediction is expected as the model tries to predict farther into the future since the scheduled admissions are more uncertain at that point. As the day in question gets nearer, more information is available on scheduled admissions, and the prediction error decreases.

The method of using the mean residual length of stay for predicting the daily discharge as presented in Section 4.4.3 was also tested for the Pediatrics unit. The corresponding results are presented in Figures 4.11, 4.12, and 4.16 together with the results of other methods.

The percentage of the absolute error by actual census is plotted for the best prediction results, using the correlation method for unscheduled admissions and the mean residual length of stay for discharges, in Figure 4.17 for one-day prediction. The maximum percentage error is less than 10% and the average percentage error is about 5% for the one-day prediction. The results indicate that the census at the Shands Teaching Hospital, Pediatrics unit can be predicted with good accuracy.

4.5.2 Results for the Gainesville Veterans Administration Hospital

Patients in all specialties at the hospital are included in this study. Data on the length of stay of patients admitted during the period of July 1974 and June 1975 were collected. The length of stay data were grouped by each specialty and by day of the week of admission. The mean length of stay and the corresponding variance are listed in Table 4.5.

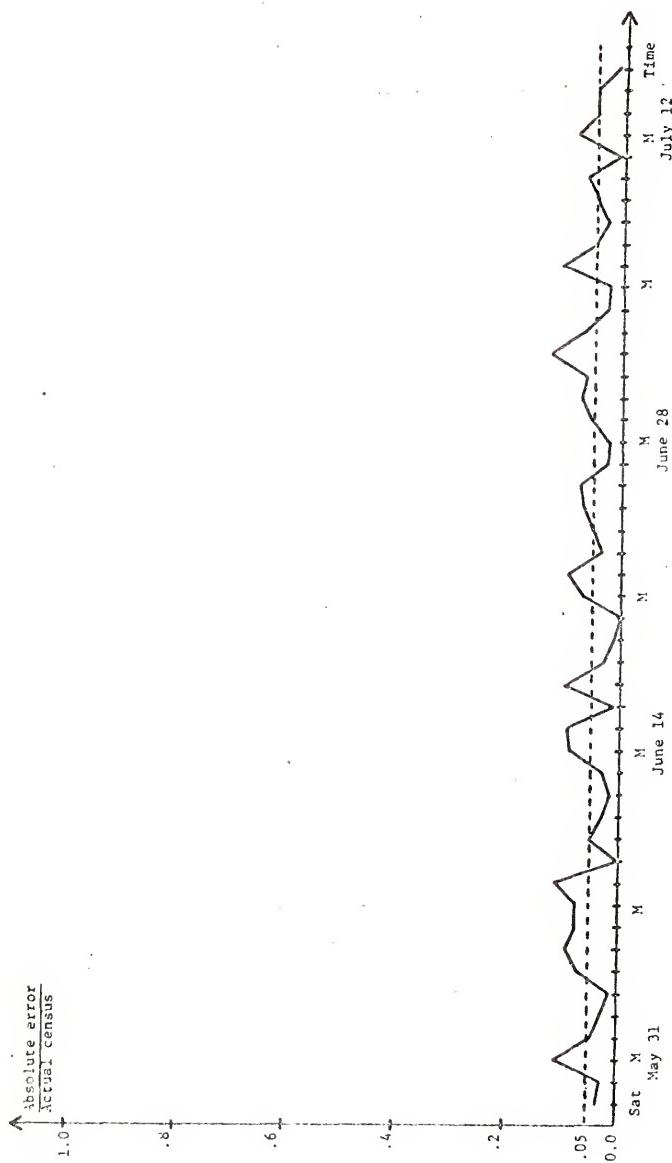


Figure 4.17. Fraction errors of one-day census prediction, Shands Teaching Hospital Pediatrics unit.

Table 4.5

Mean and Variance of Length of Stay by Service for
Gainesville Veterans Administration Hospital

Services	Total admissions	Mean (days)	Variance (days ²)
Medicine	3249	11.29	205.07
Psychiatry	933	26.73	691.15
Neurology	575	17.32	342.22
General Surgery	1232	12.24	213.16
Urology	757	9.36	174.71
Ophthalmology	413	9.76	156.62
E.N.T.	419	13.64	400.15
Orthopedics	819	14.15	324.34
Thoracic	517	17.40	275.52
Plastic	633	8.24	205.90
Neurosurgery	505	16.47	418.73
Oral Surgery	4	1.75	1.24
TOTAL	10056	13.68	310.24

The unscheduled admissions include all patients who are not listed on the schedule sheet for the admission day. Some of the patients scheduled by physicians may not report to the scheduling desk to have their reservation logged in; therefore, these patients were treated as unscheduled patients in the study. The data on the number of admissions, scheduled and unscheduled, were collected from April through August 1976. The mean value and variance of the unscheduled arrivals are listed in Table 4.6 by day of the week of admission. A weekly pattern for the unscheduled admissions can be easily recognized.

The census prediction model using the mean number of unscheduled admissions and the conditional discharge probability function was tested for the hospital. Results are plotted in Figures 4.18 and 4.19 for one-day and seven-day predictions, respectively. The corresponding frequencies of prediction errors are presented in Figures 4.20 and 4.21.

The mean error of the one-day prediction is -.77 patient with a standard deviation of 8.73 patients. The empirical census prediction errors and the corresponding percentage of prediction whose error does not exceed the bounds of error interval for Gainesville Veterans Administration are listed in Table 4.7 for one-day census prediction. It could be seen that although the prediction errors were up to ± 20 patients, this occurred less than 5% of the time.

The mean error and standard deviation for the seven-day prediction are expected to be greater than those of the one-day prediction due to the longer prediction period. The mean error and standard deviation are found to be -3.48 patients and 11.15 patients, respectively.

The census is under-predicted for both one-day and seven-day predictions. The cause of the under-prediction can be analyzed through the admission

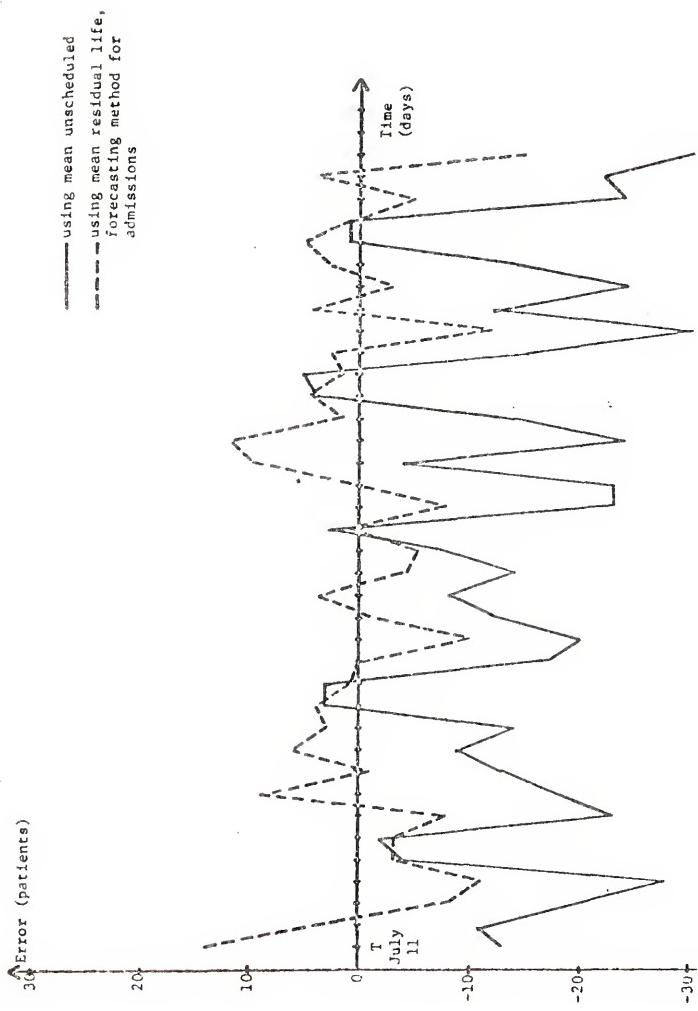


Figure 4.18. One-day census prediction errors, Gainesville Veterans Administration Hospital. (Error = predicted-actual.)

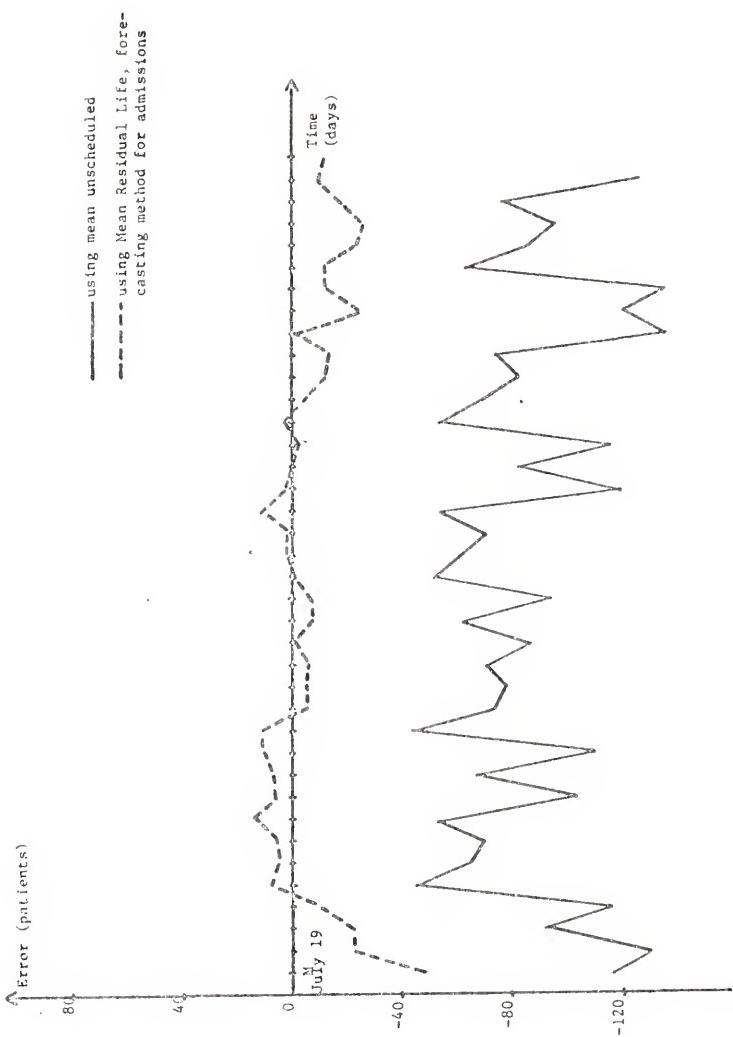


Figure 4.19. Seven-day census prediction errors, Gainesville Veterans Administration Hospital. (Error = predicted-actual.)

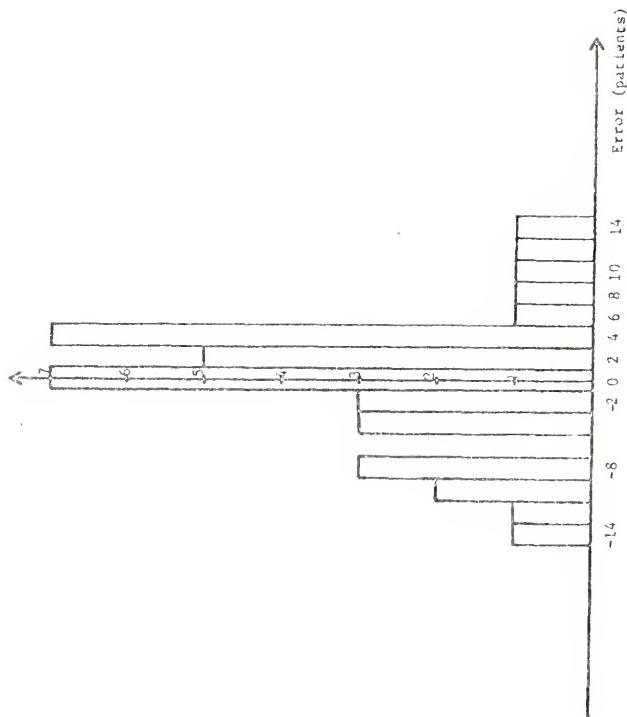


Figure 4.20. Frequency of one-day census prediction errors, Gainesville Veterans Administration Hospital.

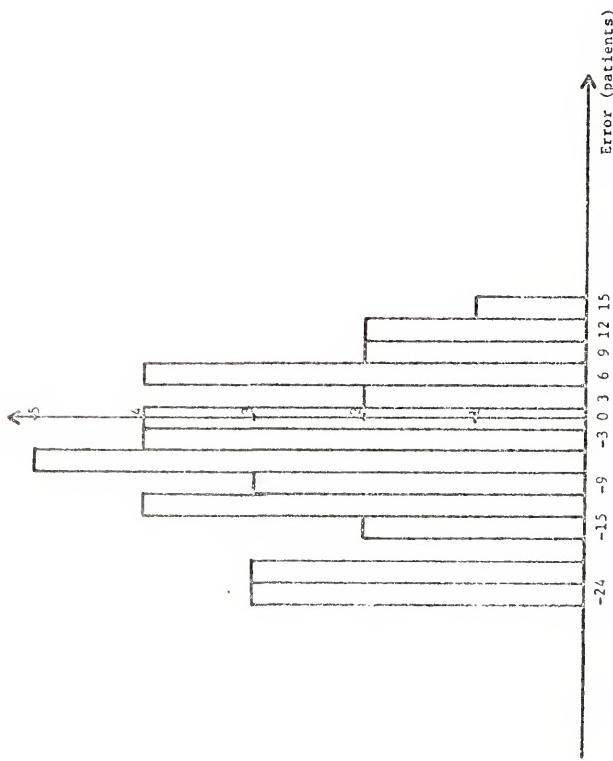


Figure 4.21. Frequency of seven-day census prediction errors, Gainesville Veterans Administration Hospital.

Table 4.6

Mean and Variance of Unscheduled Admissions by Day
of the Week for Gainesville Veterans Administartion Hospital

Day of the week	Mean	Variance
Monday	20.31	433.05
Tuesday	23.92	579.10
Wednesday	22.45	493.24
Thursday	19.15	372.34
Friday	15.83	251.60
Saturday	7.00	54.46
Sunday	7.17	53.46

Table 4.7

The Empirical Prediction Errors for
Gainesville Veterans Administration Hospital

Empirical prediction errors (patients)	Percent of prediction whose error is within the intervals
(-5, +5)	67.57
(-8, +8)	78.38
(-10, +10)	86.49
(-12, +12)	94.59
(-14, +14)	97.30

and discharge predictions. The errors in admission and discharge predictions are plotted in Figures 4.22 and 4.23, respectively. The discharge prediction error appeared to be the one that most affects the census prediction error.

The method for forecasting the number of unscheduled admissions presented in Section 4.3 was used to improve the admission estimates. The results are compared to those of the mean values by day of the week in Figure 4.22. The two results were very close for the five-week test period. However, the forecasting method would likely give a better prediction for a longer test period where changes in admissions can occur.

The method of mean residual length of stay used to predict the daily discharge in the model. The results are presented in Figure 4.23, with the mean and variance of prediction error.

The best census prediction was found with the forecasting of unscheduled admissions and the mean residual length of stay. The mean and standard deviation of the prediction errors were .43 patient and 6.4 patients for the one-day prediction, and .56 patient and 9.5 patients for the seven-day prediction. The percentage of absolute error by actual census is plotted in Figures 4.24 and 4.25. It can be seen that the maximum error was never greater than 4% and the average error was about 2% for both one-day and seven-day predictions. This error, translated into the number of patients using the total 480 beds in the hospital, is 20 patients for the maximum error and 10 patients for the average error.

The results indicate that the census at the Gainesville Veterans Administration Hospital can be predicted more accurately than the census at the Shands Teaching Hospital, especially for a longer period of prediction. These results can be explained by the fact that scheduled

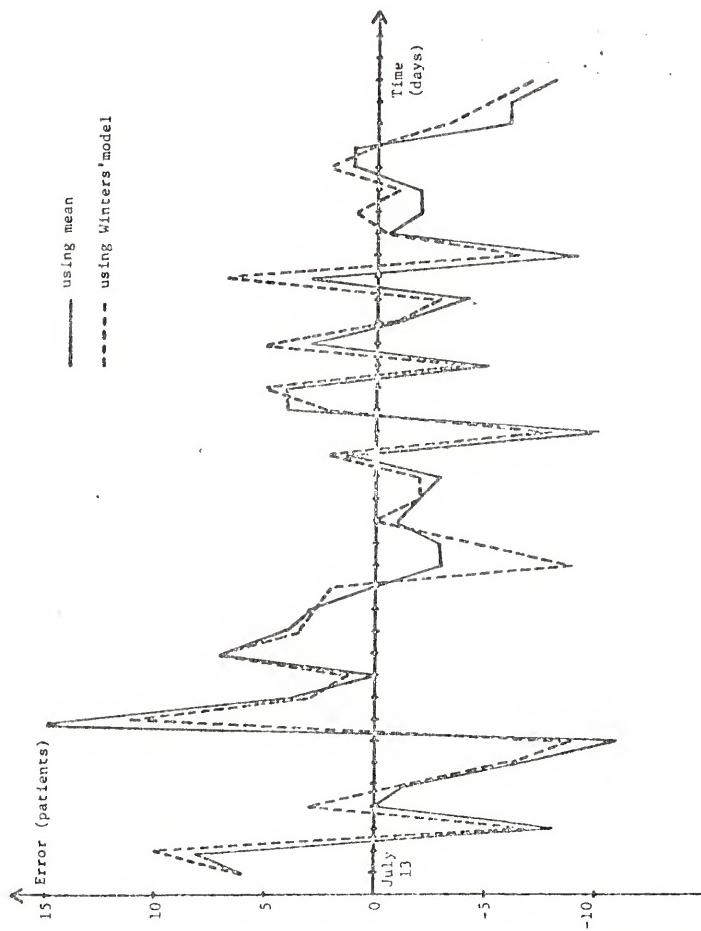


Figure 4.22. Prediction errors for unscheduled admissions, Gainesville Veterans Administration Hospital. (Error = predicted-actual.)

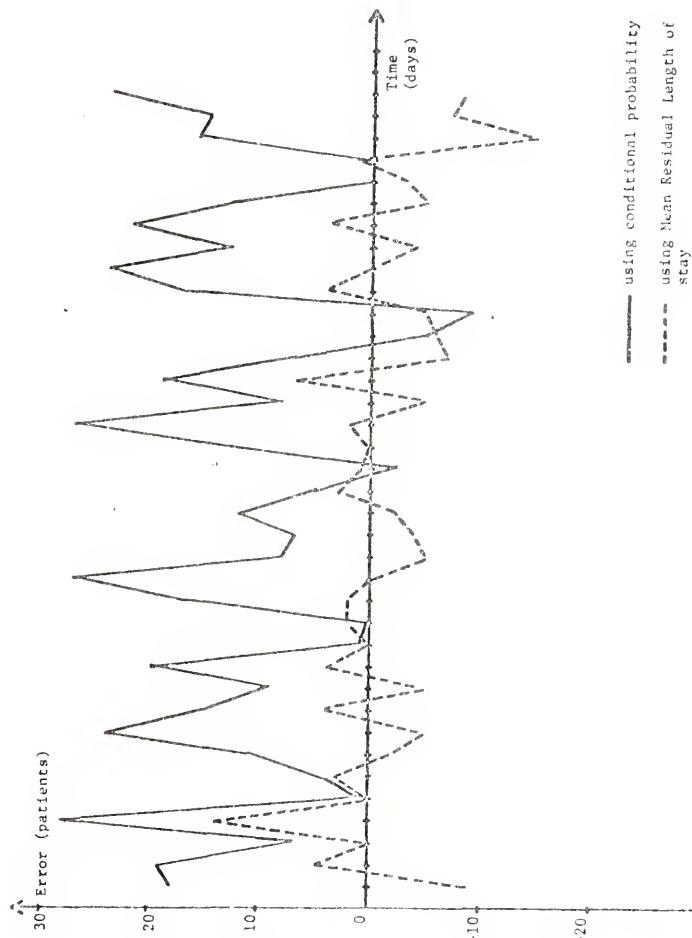


Figure 4.23. Prediction errors for discharges, Gainesville Veterans Administration Hospital.

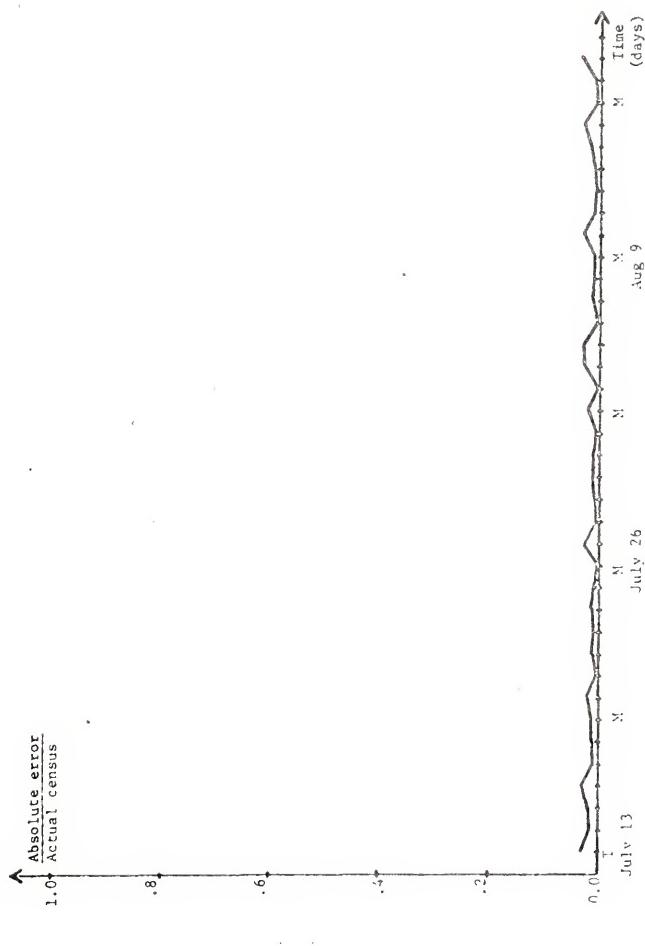


Figure 4.24. Fraction errors for one-day census prediction, Gainesville Veterans Administration Hospital.

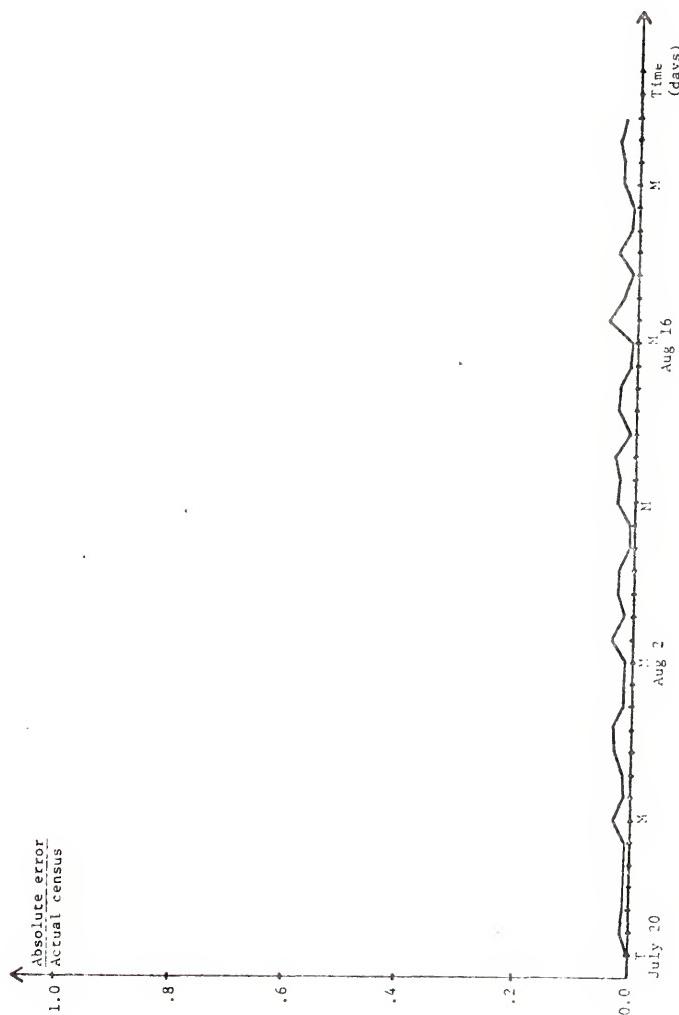


Figure 4.25. Fraction errors for seven-day census prediction, Gainesville Veterans Administration Hospital.

admissions account for a smaller portion of admissions at the V.A., thus future information has a minimal effect.

4.5.3 Results for the North Florida Regional Hospital

The North Florida Regional Hospital also operates under a referral system similar to the Shands Teaching Hospital. North Florida Regional Hospital has 170 beds for 15 different services: Medicine, Cardiovascular, Psychiatry, Dermatology, Orthopedics, Surgery, Gynecology, Urology, Neurology, Neurosurgery, Dental, E.N.T., Ophthalmology, Plastic Surgery, and Thoracic Surgery. The length of stay data were collected for the year 1975. The numbers of emergency, urgent, and direct admissions were also obtained for the same period. The mean values of unscheduled admissions are presented in Table 4.8. The census prediction process was carried out for a period in January and February 1977 for North Florida Regional Hospital. This was the peak period of the year for the hospital. The hospital was operating in a high probability of an overflow condition, the unscheduled admissions were primarily emergency admitted through the emergency room. During this period of the study, many scheduled admissions had to be cancelled and rescheduled for a later date and emergency patients were admitted to other institutions. The census prediction model was able to predict overflow situations one or two days in advance. No admission controls were applied until the admission day, thus the model did not have accurate input information to provide good predictions for a longer period in advance. Efforts are being made to obtain up to date input information of rescheduling admissions for the early part of the week. For Thursday and Friday, there are much fewer cases of rescheduling, therefore the census predictions for four or five days in advance should be valid.

Table 4.8

Mean and Variance of Emergency Admissions by Day
of the Week for North Florida Regional Hospital

Day of the week	Mean	Variance
Sunday	4.25	6.19
Monday	6.52	7.52
Tuesday	5.57	9.01
Wednesday	5.15	9.86
Thursday	5.44	6.75
Friday	5.98	10.48
Saturday	4.50	5.52

The census prediction model provides the admitting personnel of North Florida Regional Hospital information that can be based on for controlling admissions in advance instead of on the admission day. Thus, the cancellation and rescheduling problem should be eliminated.

4.5.4 Results for the Alachua General Hospital

The Alachua General Hospital is a community hospital with a bed capacity much higher than the demand for beds. The hospital is interested in a census prediction process for the purpose of staffing. There are seven main services in the hospital: Medicine, Surgery, Psychiatry, Orthopedics, Neurology, Obstetrics, and Gynecology. The length of stay data were collected by service and by the day of the week of admission from October 1975 to September 1976. The unscheduled admissions were also collected for the same period. The unscheduled admissions consist largely of emergency patients, a smaller portion of urgent patients, and a negligible portion of direct patients. The daily admissions of Alachua General Hospital have 50% or more unscheduled admissions. The unscheduled admissions of Alachua General Hospital indicate a strong weekly pattern as shown in Table 4.9.

The census prediction process was applied for all services at Alachua General Hospital excluding Psychiatry and Obstetrics. Psychiatry usually has a stable population (in the statistical sense) and is completely segregated from any other services, that is, there can be no mixing of Psychiatric patients and other patients. The admissions of obstetric patients are largely unschedulable. Therefore no admissions scheduling policy can be applied to this service, and census prediction information would be of little interest to these two services. At first, the census

Table 4.9

Mean and Variance of Unscheduled Admissions
by Day of the Week for Alachua General Hospital

Day of the week	Mean	Variance
Monday	23.44	26.79
Tuesday	22.87	22.96
Wednesday	18.85	24.92
Thursday	17.13	21.28
Friday	14.12	14.16
Saturday	9.85	9.98
Sunday	15.12	19.99

prediction process was tested for a period of two weeks at the beginning of December 1976. It was recognized that the Christmas Holidays were beginning to have effects on the census levels; therefore, the prediction process was halted until the beginning of January 1977. The census prediction model gives the expected census levels and the lower and upper bounds on a 95% confidence intervals for up to 15 days in advance. The results for one-day census prediction are plotted against the actual census levels for the period from January 5 to February 1977 in Figure 4.26. Better prediction results were obtained by using the mean residual length of stay for discharge prediction. The mean prediction error was .911 patient with a 6.48 patient standard deviation. The prediction errors are plotted in Figure 4.27. From Figure 4.26, the prediction of census levels for the second week of the prediction period was much lower than the actual census levels. This high discrepancy could be partly due to the behavior of admissions and discharges to unusually cold weather in the Gainesville area. This effect can be seen more pronouncedly in the seven-day census prediction results in Figure 4.28. The fraction errors of the one-day and seven-day census prediction are illustrated in Figures 4.29 and 4.30. The average fraction errors were 0.027 and 0.063 for one-day and seven-day prediction, respectively. The prediction results indicate that the model under-predicts for long range prediction periods. This under-prediction resulted from future information on scheduled admissions not being available far in advance of the admission date. The under-prediction error was reduced as the admission date got closer and more scheduled information was available. The seven-day prediction results of Alachua General Hospital were better than those of Shands Teaching Hospital since the scheduled admissions of Alachua General Hospital were a much

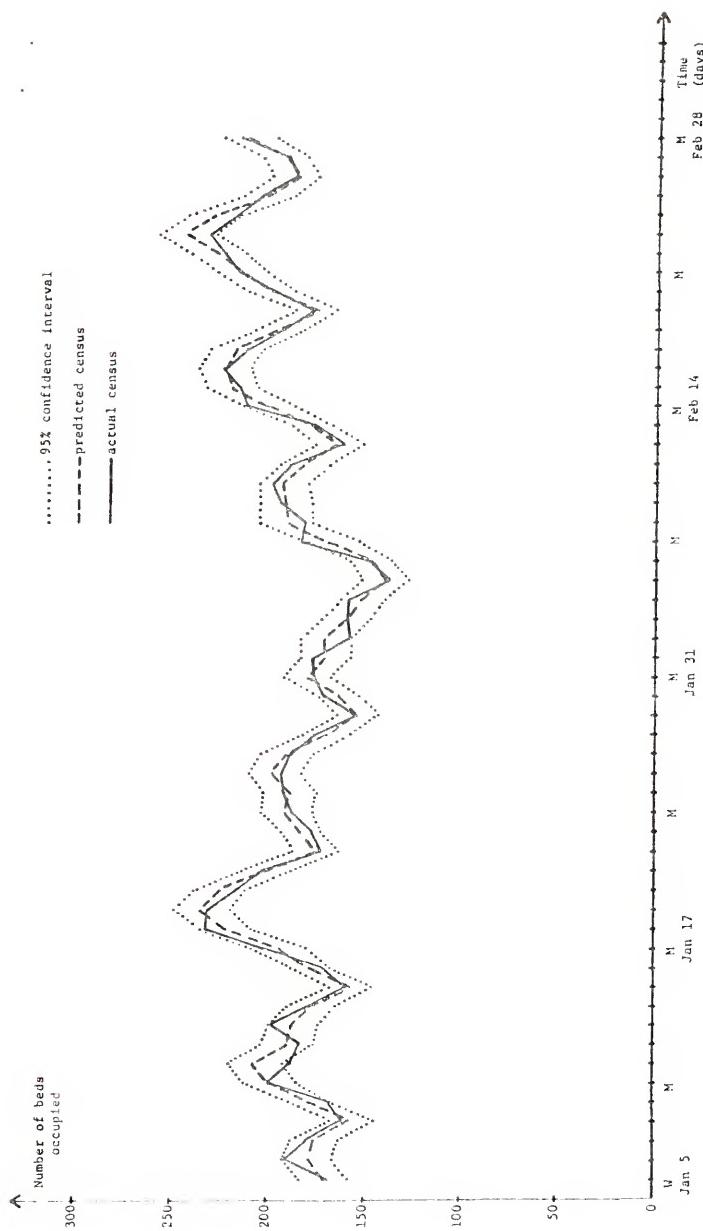


Figure 4.26 One day census prediction, Alachua General Hospital.

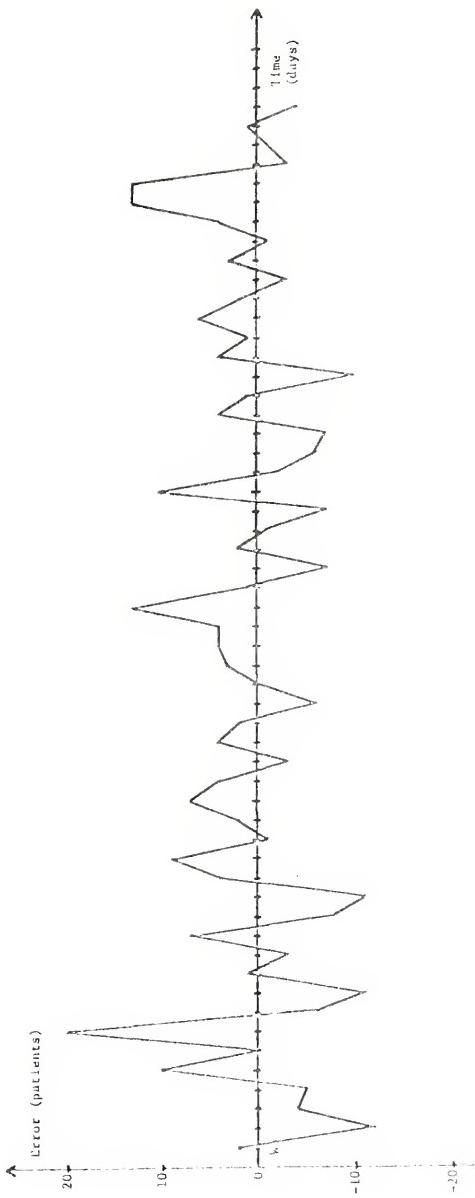


Figure 4.27 One-Day census prediction errors, Alachua General Hospital.
(Error = predicted-actual.)

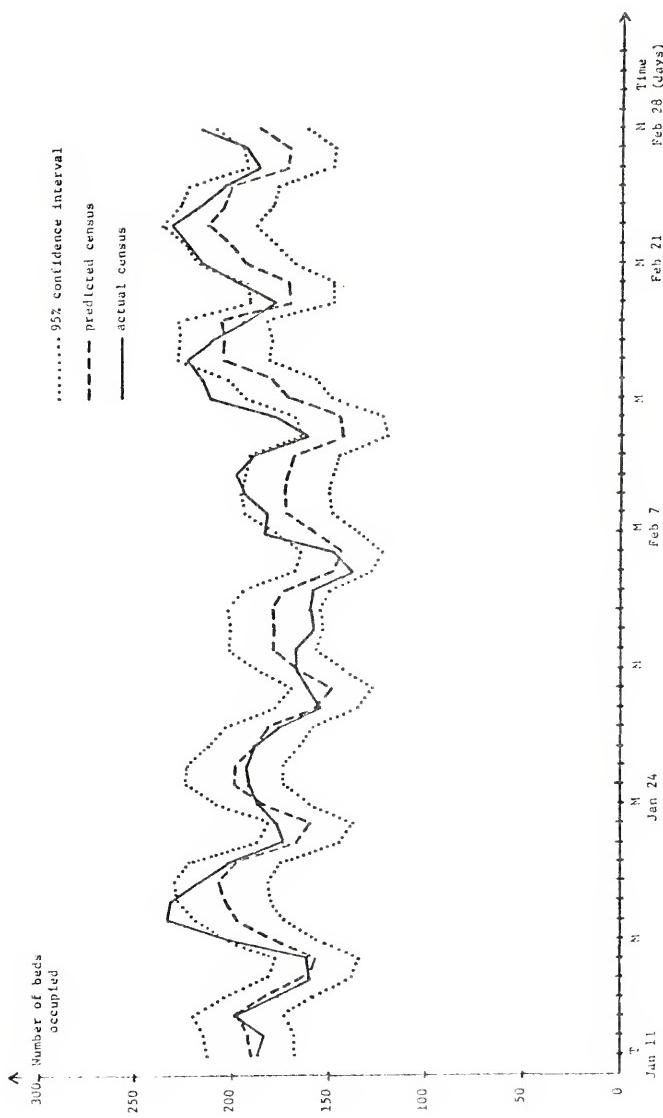


Figure 4.28 Seven-Day census prediction, Alachua General Hospital.

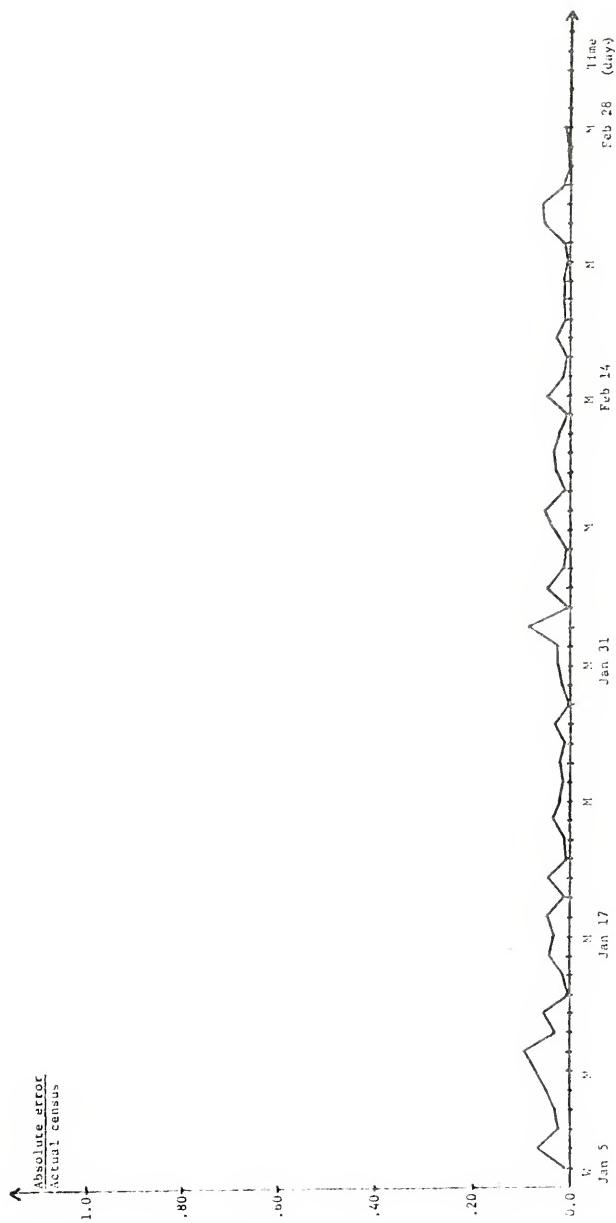


Figure 4.29 Fraction errors of one-day census prediction, Alachua General Hospital.

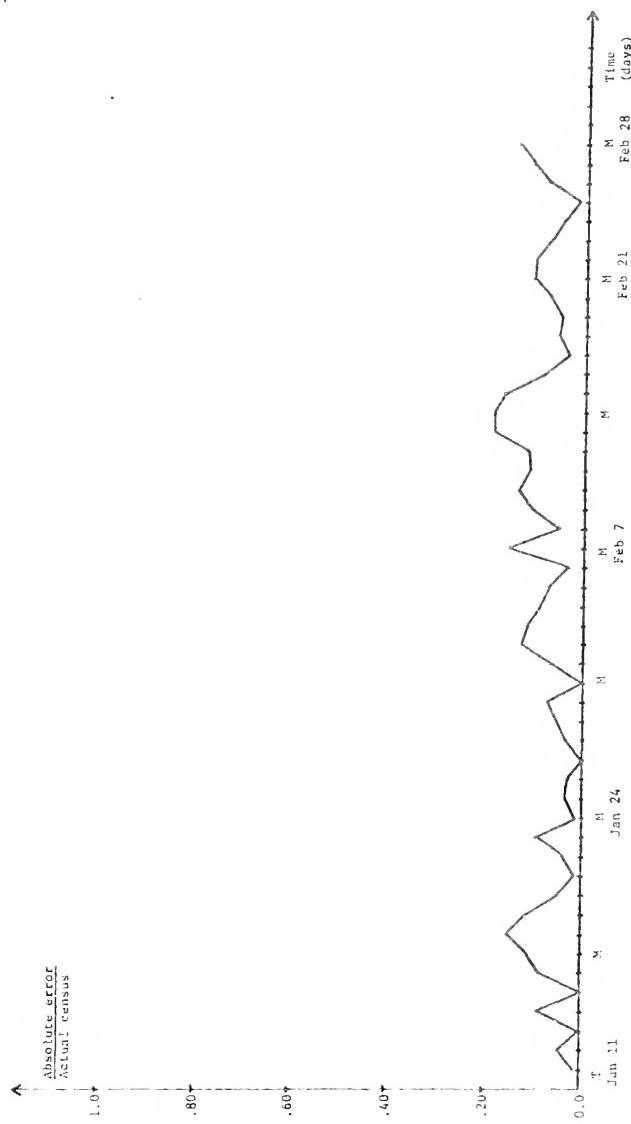


Figure 4.30 Fraction errors of seven-day census prediction, Alachua General Hospital.

smaller portion of admissions than for Shands Teaching Hospital. Future information on scheduled admissions therefore did not affect the prediction as much as at Shands Teaching Hospital where almost all admissions were scheduled.

A summary of results of the census prediction for the four hospitals in the study is presented in Table 4.10.

4.6 Characteristics of the Hospitals in the Study

This study covered a wide range of hospitals: a teaching hospital, a Veterans Administration hospital, a proprietary hospital, and a community general hospital. Each hospital has its own operating setting which creates different effects on the census prediction process. In this section, the characteristics of the hospitals which have a significant influence on the prediction results will be identified.

The two main components in the census prediction process, as presented in Section 4.2, are the admission and discharge processes. Another hidden element is the patient population which can affect the length of stay and the arrival process. The size of the hospital or unit used for the census prediction process also influences the prediction results. These four components will be discussed in the following sections.

4.6.1 The Admissions Process

Admissions policies vary among the hospitals. In the Shands Teaching Hospital, most admissions are prescheduled based on physicians' referrals. The number of daily scheduled admissions is controlled by the ward physician in some services, and by a quota (maximum limit) imposed on other services. In the Pediatrics unit, all scheduled admissions are centralized through a ward secretary who takes requests for admissions from physicians

Table 4.10
Summary of Census Prediction Results

Hospitals	Size	Average utilization	Average prediction errors (patients)	Standard deviation of errors	Average fraction error
Shands Teaching Hospital (Pediatrics Unit)	92	80%	-1.018	4.347	5%
Gainesville V.A. Hospital	480	85%	.43	6.4	2%
North Florida Regional Hospital	170	96%	Not available		
Alachua General Hospital	300*	60%	.911	6.48	2.7%

*The number of beds of Alachua General Hospital is an approximate number, since the hospital is overbedded and closes up many sections at different times.

and can ask them to review scheduled dates when too many admissions are requested for one day. The Pediatrics ward secretary can be considered as a scheduling mechanism who bases her decisions entirely upon personal working experience. A small percentage, 15%-20%, of the admissions are unscheduled patients and cannot be controlled by any means.

Unscheduled admissions at the Gainesville Veterans Administration account for more than 50% of the daily admissions. The unscheduled admissions are attributed to walk-in patients, and emergency or urgent patients from outpatient clinics or ambulatory care. The daily scheduled admissions are controlled entirely by physicians who select a convenient date for patients as well as for themselves, and sometimes Radiology service and operating room schedules. The admissions control at the Gainesville Veterans Administration is completely decentralized without a set procedure for prescheduling patients.

The North Florida Regional Hospital operates under a referral system like the Shands Teaching Hospital. The bulk of the daily admissions are scheduled patients. All requests for admissions are centralized at the admissions office which does not manipulate the requested date unless the maximum capacity of the hospital is reached within one day of the scheduled date. This policy results in much rescheduling on the admission day to handle the overflow problem. The unscheduled patients are primarily emergency patients who are referred by their physicians through the emergency room of the hospital.

Alachua General Hospital has more than 50% of the daily admissions as unscheduled admissions. Since the hospital is overbedded all requests for admission are granted and no control is imposed on admissions. The assignment of the nursing staff to handle the patient load is flexible and

can be arranged within 24 hours to take care of additional load when necessary.

In summary, the four Gainesville hospitals can be categorized in two groups by the characteristics of admissions. For Shands Teaching Hospital and North Florida Regional Hospital, admissions are primarily scheduled, the unscheduled admissions account for a small number with a low standard deviation. Therefore the census prediction results are expected to have a small deviation from the actual census levels. However, future information on scheduled admissions causes the census prediction errors to increase for predictions further into the future. The seven-day census prediction results have larger uncertainty than the one-day predictions. For Gainesville Veterans Administration Hospital and Alachua General Hospital, the unscheduled admissions account for the bulk part of admissions. The census prediction results have a large variance or a high uncertainty. The future information on scheduled admissions does not have much influence on the long range prediction. The uncertainty on census prediction for more advanced prediction does not increase as much as those for Shands Teaching Hospital or North Florida Regional Hospital.

4.6.2 The Discharge Process

Physicians are usually required to estimate the length of stay of their patients, but this requirement is seldom enforced in most hospitals and the quality of these estimates varies widely between hospitals. The discharge prediction used here is therefore based completely on historical data from each hospital. The uncertainty of the discharge prediction might be reduced by using the updated physicians' estimate on the patient length of stay.

The weekend effect is more pronounced for a hospital with a longer average length of stay. The prediction of daily discharge can follow the weekly pattern, thus giving better prediction.

4.6.3 The Patient Population

The type of patients can also influence the prediction of admissions and discharges. For the Gainesville Veterans Administration Hospital, the patients are from a relatively known population: the veterans who reside in North Florida and South Georgia. Therefore, the prediction of unscheduled admissions is better than expected; that is, the standard deviation of the prediction errors is only half of the standard deviation of the historical data. For a large percentage of the time, admissions are predicted within an error interval equivalent to half of the variation range of past data. For Alachua General Hospital, the unscheduled admissions come from a relatively unknown population. The predictions on admissions have larger errors than those for the V.A. Hospital. Moreover, the Gainesville Veterans Administration Hospital has strict policy on the "no show" patients, the scheduled admissions can be considered as the actual admissions. For other hospitals, "no show" patients can be a significant portion of the admissions and can affect the prediction strongly. For Shands Teaching Hospital Pediatrics unit, the "no show" patients were considered as negative unscheduled admissions in the prediction of admissions.

The type of patients can also influence the discharge prediction. For a population of critical patients as at Shands Teaching Hospital, the lengths of stay are usually longer and with a higher variation. Thus the prediction of discharges involves a higher uncertainty.

4.6.4 The Size of the Hospital

The size of the hospital appears to have a significant effect on the census prediction. If one considers certain units within a hospital, the census prediction errors for these units are relatively larger than the errors for the whole hospital. The expected value and variance of census are evaluated using the assumption that the census is normally distributed and the length of stay is statistically independent. By the central limit theorem, the normality approximation is more accurate for a large number of patients. Therefore, the larger the size of the hospital, the better the census prediction. One would expect the census prediction process for all services of Shands Teaching Hospital to have better results than the 5% error obtained for the Pediatrics unit.

A summary of hospital characteristics and influences on the census prediction is presented in Table 4.11.

4.7 The Scheduling of Elective Admissions

The census prediction process provides information on census levels at any future date. This census information can be used as an additional base for admissions decisions. In this section, some scheduling decisions using census prediction information will be presented. As discussed previously, the objectives of a hospital are to minimize census fluctuation about a desired census level. In the following discussions, these objectives are used to develop mathematical models.

4.7.1 The Underbedded Hospitals

For underbedded hospitals, the demand for beds is generally higher than bed availability. The objective of minimizing the variation in daily census is equivalent to maximizing the average census since as the census

Table 4.11

Effects of Hospital Characteristics on the Census Prediction

Characteristics	Smaller error	Larger error
Large percentage of scheduled admissions	for short range prediction	for long range prediction
Large percentage of unscheduled admissions	for long range prediction	
Relative known patient population	X	
Small percentage of 'no show'	X	
Critical patients with long length of stay		X
Large number of beds	X	

is pushed to the limit, the variation in census decreases to zero. Moreover, by maximizing the census, hospital revenue is increased, and patient's waiting time is decreased. For a well-managed admissions system, the elective patients can be scheduled so that the occupancy level is maximized while cancellation and turnaways are kept at appropriate levels.

A mathematical model can be used to express the objective of maximizing the average census level subject to the constraint limits on the chance of overflow as follows

$$\text{Max } \sum_{t=1}^T E[c_t | e_t]$$

$$\text{s.t. } P[c_t \geq B | e_t] \leq \alpha \quad t = 1, \dots, T$$

where c_t = the census of day t ,

$e_t = (e_1, e_2, \dots, e_t, 0, 0, \dots)$ = decisions on the number of scheduled patients,

T = the planning horizon,

B = bed capacity, and

α = the acceptable level of overflow probability.

Assuming the probability density function $f(c_t)$ of the census level can be approximated by a normal distribution, the constraint can be rewritten as

$$E[c_t | e_t] + z_{\alpha/2} (V[c_t | e_t])^{1/2} \leq B$$

In order to achieve a solution to the problem, Swain (62) has observed that it is never optimal to leave a bed empty, if it can be filled without violating the chance constraints. Thus, the problem becomes a chain of

simpler constrained problems. At each stage, the problem is to maximize the number of elective admissions subject to the appropriate overflow constraints, i.e.,

$$\text{Max } e_i$$

$$\text{s.t. } E[c_t | e_i] + z_{\alpha/2} (V[c_t | e_i])^{1/2} \leq B.$$

4.7.2 The Overbedded Hospitals

For an overbedded hospital, maximizing the average daily census is not necessarily equivalent to minimizing the variation in daily census. Efforts can be made to achieve maximum occupancy capacity for some days, but for other days the census levels may fall well below the maximum capacity due to low demand for beds. Thus, the maximization of average daily census, creates a high fluctuation in the daily census and with staff and resources to handle peak periods, hospital resources are under-utilized a large percentage of time. Minimizing variation in the daily census is a more appropriate objective for the overbedded hospital. The problem of minimizing the variation in daily census can be expressed mathematically as

$$\text{Min } \sum_{t=1}^T V[c_t | e_t]$$

$$\text{s.t. } P[c_t \leq A | e_t] \leq \alpha \quad t = 1, 2, \dots, T,$$

where A is the minimum level of census that the hospital is willing to accept. For the moment A is assumed to be defined.

One way to look at the problem of minimizing variation in the daily census is to determine an operating census level for the hospital. The problem is then reduced to maximizing the occupancy level subject to

constraint limits on the chance of going over the operating levels. That is, the problem becomes the one of underbedded hospital with maximum occupancy levels at the operating levels. The difficulty here is the determination of the operating level. If it is too low, then the hospital is operating below the effective level, and the waiting time for patients becomes too long. If it is too high, the variation in daily census cannot be improved. The average occupancy level can be used as a trial level at first. Waiting time constraints can be imposed on the problem to insure a long admissions delay will not occur. The problem can be expressed mathematically as

$$\begin{aligned} \text{Max } & \sum_{t=1}^T E[c_t | e_t] \\ \text{s.t. } & P[c_t \geq Q | e_t] \leq \alpha \quad t = 1, \dots, T \\ & e_1 + e_2 + e_3 + e_4 \geq r_1 \\ & e_1 + e_2 + e_3 + e_4 + e_5 \geq r_1 + r_2 \\ & \sum_{i=1}^{j+B} e_i \geq \sum_{i=1}^j r_i \end{aligned}$$

where Q is the operating level.

The structure of this problem can produce an infeasible solution however; for example, the demand for admissions can increase so greatly that the hospital is unable to handle all patients at the existing operating level without delaying admissions for more than three days. The census level and the waiting queue have to be constantly reviewed so that a new operating level can be recognized, and plans for staff and resources can be made.

4.7.3 The Stochastic Demand

In the two previous sections, it is assumed that all the decisions on the number of scheduled admissions can be satisfied, i.e., there always exists a "queue" of patients for admissions. In a practical situation, this assumption does not necessarily hold, especially for overbedded hospitals. Barber (7) studied the problem of achieving the objectives when the demand is stochastic. Instead of looking at the decision variables e_i , the number of admissions for day i , Barber introduced s_i , the number of actual admissions for day i . The relation between e_i and s_i is as follows

$$s_i = e_i \quad \text{for requests more than availability}$$

$$s_i < e_i \quad \text{for requests less than availability.}$$

Barber assumed that the probability distribution $f[s_t | e_t]$ is known. The same solution for the problem is found for the case of separable distribution, i.e.,

$$f[s_t | e_t] = f[s_1 | e_1] f[s_2 | e_2] f[s_3 | e_3] \dots f[s_t | e_t]$$

A more complicated solution is presented for non-separable distribution in his dissertation (7).

In the case of overbedded hospitals, the stochastic demand introduces much more complication to the minimization of variation in the daily census. The two equivalent formulations presented in Section 4.7.2 can be used to approximate the solution for the case of separable distribution $f[s_t | e_t]$.

Since my research focuses on the census prediction providing additional information to help in deciding the number of scheduled admissions, it tends to leave the decisions up to administrative personnel

instead of developing rigid scheduling rules. It is expected that with the implementation of the census prediction model, administrative personnel can develop skills through experience over time.

CHAPTER FIVE

FURTHER RESEARCH

This research presented models for the allocation of hospital beds between services and models for the prediction and control of inpatient admissions. These models develop insights into the process of bed allocation and patient admissions which can be used to improve the operation of the hospital.

The bed allocation models described in this research provide hospital decision-makers with a means for distributing beds among services within the hospital to reduce the cost of inefficient resource allocation. The four models presented vary from simple to complex models, which can be chosen to describe real world hospital systems at various degrees of complexity. The queueing models--Poisson arrival process and Erlang-k service time distribution with a random number of servers--derived for bed allocation are of theoretical value and can be applied to other problems besides the hospital admissions problem. In this research the optimization problem of bed allocation was solved by a heuristic algorithm using a search method about neighboring allocations. As the numbers of beds and services increase, other search methods should be explored to find one which provides a faster convergence. An extension of this research would be investigation of different random search techniques for their efficiency.

The census prediction model gives the expected census levels for future dates based on currently known information of the midnight census,

the elective admissions scheduled for any future date, and historical data on length of stay and unscheduled admissions. The census prediction model provides the hospital admitting office with information to use in deciding the number of additional elective admissions to be scheduled for each future day. Different methods for predicting admissions and discharges used in the census prediction model were presented. The expected unscheduled admissions could be found by using the correlation between unscheduled and scheduled admissions or by a time series model--Winters' seasonality model. Other time series models could be explored for predicting unscheduled admissions.

McClave and Marks (46) present a time series regression model to identify and estimate models for the residual series. An extension of the use of this regression model can be applied to the unscheduled admissions and also the census prediction residuals. The implementation phase of the census model continues at various hospitals: Shands Teaching Hospital, Alachua General Hospital, and North Florida Regional Hospital. Data on length of stay and unscheduled admissions can be gathered in parallel with the prediction and can be used to update the model as often as required. Moreover, when sufficient observations of census prediction results are collected, time series models can be tested on the census prediction errors to reduce error on census prediction.

In this research, effort has been concentrated on improving the census prediction results. The next step would be to study admissions scheduling policies based on the predicted census. Barber (7) presented a method of scheduling admissions which incorporated the stochastic nature of requests for admissions. Barber considered scheduling policies to maximize occupancy levels relative to constraints on overflow probab-

ility. Maximization of occupancy levels may not be the answer for overbedded hospitals where fluctuations in census would force the hospital to maintain staff and resource at levels close to peak demand. Thus, staff and resources are under-utilized for a large percentage of time. Other objectives for admissions control should be considered such as minimization of variation in occupancy levels for overbedded hospital. An extension of Barber's stochastic admissions policies to minimize variation in occupancy levels is a worthwhile area to investigate. In practice, overbedded hospitals tend to accept as many requests for admissions as they get and no control is exercised on admissions. Therefore, these hospitals usually maintain staff at a much higher level than the average operating level. Staff assignment is flexible so that rearrangement of nursing staff can be done in a relatively short time to handle peak demand. With scheduling policies to minimize variation in census, the hospital may be able to operate efficiently at a lower staffing level. Simulation models can be applied using different scheduling policies--for maximizing occupancy levels and for minimizing variation in occupancy levels--to compare their effects on the efficiency of the hospital operation.

Another area that has not been explored is the dependence of the patients' length of stay. In this research, it has been assumed that the length of stay of all patients are statistically independent. The variance of the census prediction, therefore, was found as the sum of the variances of various components as in Equation (4.10). It has been recognized that prediction errors decrease relatively as the size of the hospital increases. Moreover, the census prediction of the whole hospital is usually much better than that of individual services.

Therefore, study on the correlation among the services or patients' length of stay may reveal interesting results.

All these extensions presented would make the study of hospital admissions process more complete.

APPENDIX A

THE $M/E_k/s_j/s_i^*$ QUEUEING SYSTEM

The relationship between state probabilities $P*(m)$ of the queueing system $M/E_k/s_j/s_i^*$ (Poisson arrivals to s_j continuously available servers plus s_i servers available on a random basis, each of the s_j+s_i servers possessing the same Erlang k service time distribution, and no waiting line) can be found by solving a set of equations representing the steady state of the probability system (known as Chapman-Kolmogorov equation). The state-transition-rate diagram is commonly used as a means of displaying the system in order to find the set of equations. In such a diagram, state m is represented by a circle surrounding the number m . The branches identify the permitted transitions and the branch labels give the transition rates. The $M/E_2/1/1^*$ and the $M/E_2/1/2^*$ systems are represented by diagrams in Figures A.1 and A.2, respectively. The state of the system is defined as an n -tuple, where n is the number of customers in the system, and the individual entries indicate the stage of service of the individual customers. For example, the state (12) defines a state with two customers, one in stage 1 of service, the other in stage 2 of service.

The Chapman-Kolmogorov equations can be written directly from the diagrams by treating the diagrams as a network. This is done by simply treating probabilities as node pressures, transition rates as pipe capacities, and writing the node balance equations. For example, the equations for the $M/E_2/1/1^*$ system (Figure A.1) are as follows.

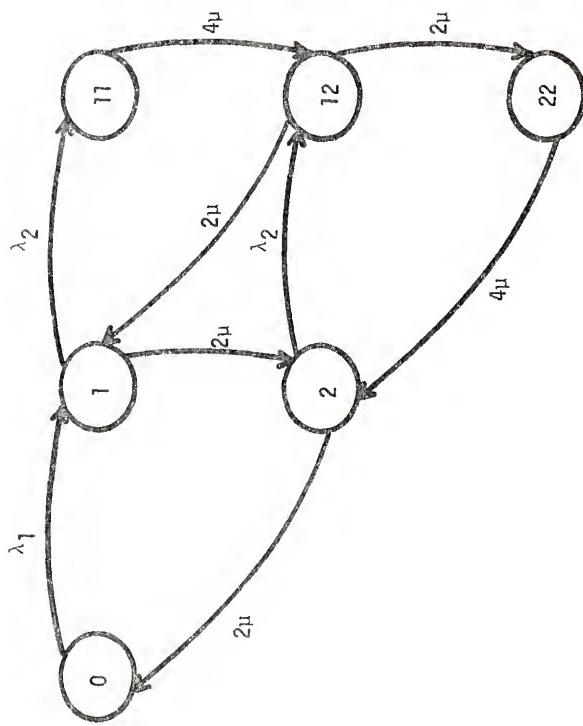


Figure A.1 The state-transition-rate diagram for the $N/E_2/1/1^*$ system.

$$\lambda_1 p_0 = 2\mu p_2$$

$$(\lambda_2 + 2\mu) p_1 = \lambda_1 p_0 + 2\mu p_{12}$$

$$(\lambda_2 + 2\mu) p_2 = 2\mu p_1 + 4\mu p_{22}$$

$$4\mu p_{11} = \lambda_2 p_1$$

$$4\mu p_{12} = \lambda_2 p_2 + 4\mu p_{11}$$

$$4\mu p_{22} = 2\mu p_{12}$$

The relationships among the state probabilities are found from these equations to be

$$p_1 = p_2 = \frac{\lambda_1}{2\mu} p_0$$

$$p_{11} = p_{22} = \frac{\lambda_1 \lambda_2}{8\mu^2}$$

$$p_{12} = \frac{\lambda_1 \lambda_2}{4\mu^2}$$

Thus, the relationships among the state probabilities $P^*(m)$ are

$$P^*(1) = p_1 + p_2 = \frac{\lambda_1}{\mu} P^*(0)$$

$$P^*(2) = p_{11} + p_{12} + p_{22} = \frac{\lambda_1 \lambda_2}{2\mu^2} P^*(0)$$

$$= \frac{\lambda_2}{\mu} \frac{1}{2} P^*(1)$$

Similarly, the equations for the $M/E_2/1/2^*$ system can be written, using Figure A.2, as

$$\lambda_1 p_0 = 2\mu p_2$$

$$(\lambda_2 + 2\mu) p_1 = \lambda_1 p_0 + 2\mu p_{12}$$

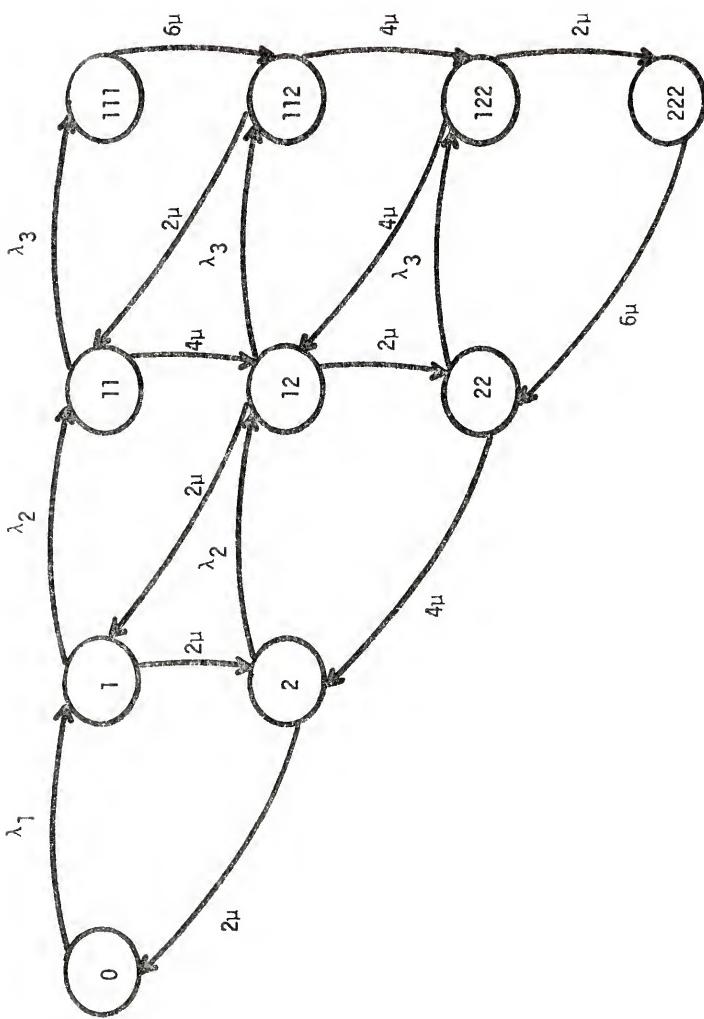


Figure A.2 The state-transition-rate diagram for the $M/E_2/1/2^*$ system.

$$(\lambda_2 + 2\mu)p_2 = 2\mu p_1 + 4\mu p_{22}$$

$$(\lambda_3 + 4\mu)p_{11} = 2\mu p_{112} + \lambda_2 p_1$$

$$(\lambda_3 + 4\mu)p_{12} = 4\mu p_{11} + 4\mu p_{122} + \lambda_2 p_2$$

$$(\lambda_3 + 4\mu)p_{22} = 2\mu p_{12} + 6\mu p_{222}$$

$$6\mu p_{111} = \lambda_3 p_{11}$$

$$6\mu p_{112} = \lambda_3 p_{12} + 6\mu p_{11}$$

$$6\mu p_{122} = 4\mu p_{112} + \lambda_3 p_{22}$$

$$6\mu p_{222} = 2\mu p_{112}$$

Since the relationships between states 0, 1, and 2 do not change, it follows that

$$p_1 = p_2 = \frac{\lambda_1}{2\mu} p_0$$

The relationships among other state probabilities can be found from these equations to be

$$p_{11} = p_{22} = \frac{\lambda_1 \lambda_2}{8\mu^2} p_0$$

$$p_{12} = \frac{\lambda_1 \lambda_2}{4\mu^2} p_0$$

$$p_{111} = p_{222} = \frac{\lambda_3 \lambda_2 \lambda_1}{48\mu^3}$$

$$p_{112} = p_{122} = \frac{\lambda_3 \lambda_2 \lambda_1}{16\mu^3}$$

Therefore, the relationships among the state probabilities $p^*(m)$ are

$$P^*(1) = \frac{\lambda_1}{\mu} P^*(0)$$

$$\begin{aligned} P^*(2) &= p_{11} + p_{12} + p_{22} = \frac{\lambda_1 \lambda_2}{\mu^2} \frac{1}{2} P^*(0) \\ &= \frac{\lambda_2}{\mu} \frac{1}{2} P^*(1) \end{aligned}$$

$$\begin{aligned} P^*(3) &= p_{111} + p_{112} + p_{122} + p_{222} = \frac{\lambda_1 \lambda_2 \lambda_3}{\mu^3} \frac{1}{3!} P^*(0) \\ &= \frac{\lambda_3}{\mu} \frac{1}{3} P^*(2) \end{aligned}$$

The procedure can be continued for the $M/E_2/1/3*$, and in general the $M/E_2/s_j/s_i^*$ systems. The relationships among state probabilities $P^*(m)$ are

$$P^*(m) = \frac{\lambda_m}{m\mu} P^*(m-1) \quad \text{for } m=1,2,\dots,s_j+s_i$$

where $\lambda_m = \lambda$ for $m=1,2,\dots,s_j$

$\lambda_m = \lambda \phi_j^*(m-s_j)$ for $m=s_j+1,\dots,s_j+s_3$, and

$\phi_j^*(n) =$ the probability of having n central pool beds available
for patients of service j .

APPENDIX B

BLOCK DIAGRAMS AND PROGRAM LISTING FOR
A GPSS SIMULATION FOR BED ALLOCATION

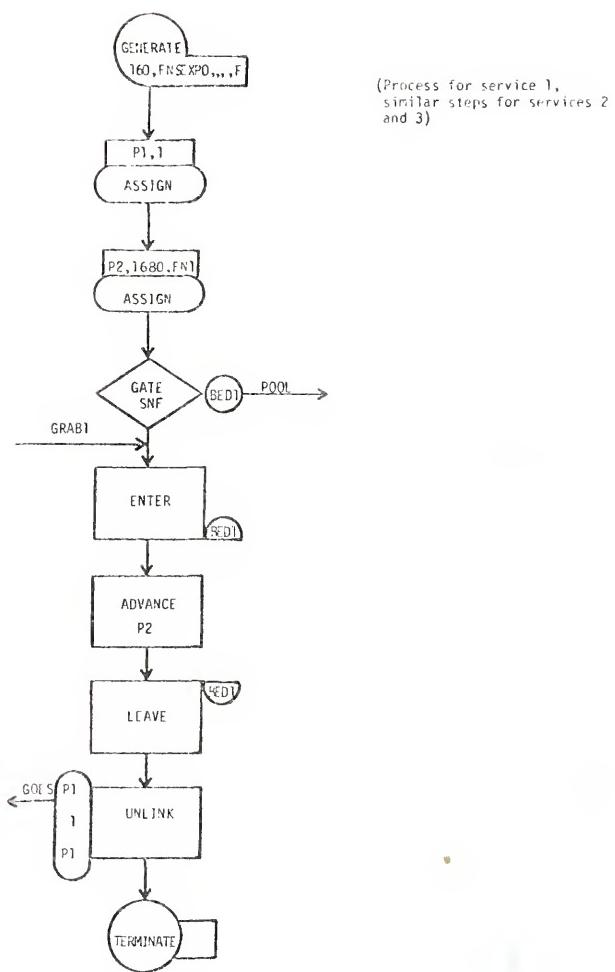


Figure B.1 Block diagrams for the GPSS simulation model on a three-service hospital.

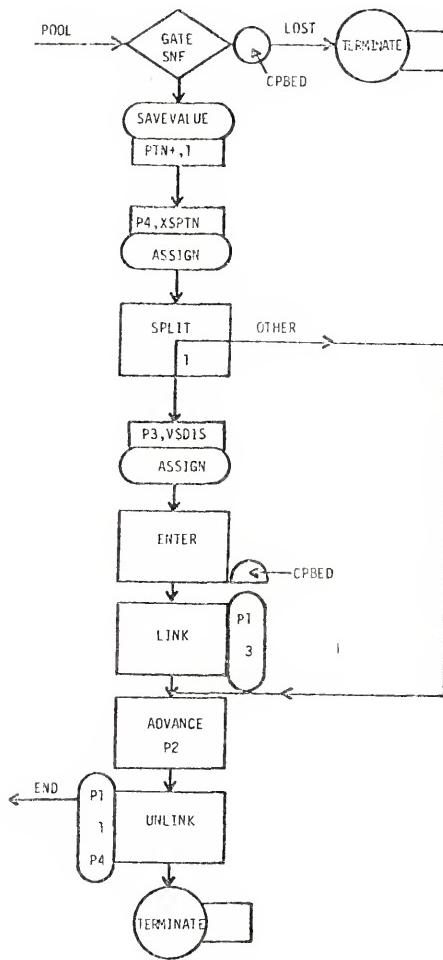


Figure B.1 (continued)

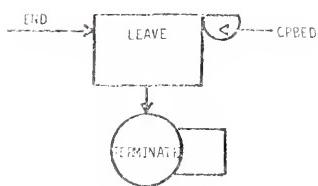
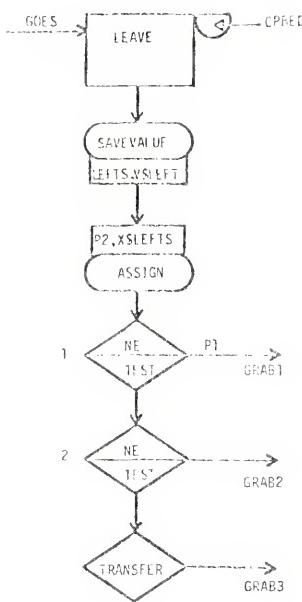


Figure B.1 (continued)

APPENDIX C
SUMMARY OF DATA COLLECTION AND DESCRIPTION

C.1 Introduction

Reports on applied research of this kind often leave the erroneous impression that all the data required to develop and test the models were readily available and all that was required of the model builder was a clever manipulation of these available data. This appendix will serve to correct that impression as it concerns this research. The appendix is also intended to serve as a guide to other potential users of the prediction models described here. Each of the hospital's data systems differed in significant ways from that of the other three test hospitals. Shands Teaching Hospital used a university computer shared with multi-campus instructional, administrative and research users, the Alachua General Hospital used a remote data link to the county's data processing center. The Gainesville Veterans Hospital had no computer services other than that provided by V.A. data processing in Austin, Texas and North Florida Regional Hospital used a G.E. time sharing system. These four environments were therefore completely diverse in their data processing architecture and capabilities. It is hoped that a potential user of the prediction system will find some insights (and some solace) in the following descriptions of the techniques used to extract and process the data required to develop the model in each disparate setting.

C.2 Shands Teaching Hospital

C.2.1 Length of Stay Data

At Shands there were two sources which could provide the length of stay of patients: The Patient Abstract and the Condensed Billing file. Neither source had the complete information that the study required. Therefore data from these two sources were merged by medical record number and the length of stay to obtain necessary information. The two sources of data are described in the following sections.

C.2.1.1 Patient Abstract

The patient discharge abstract for each admission is available from 1970 onward. The patient abstract tape provides information on the length of stay, discharge diagnoses, attending physicians, operations, hospital service and demographic data on patients such as age, sex, race, and economic class. The patient abstract tape only has the month and year of discharge date and no information on admission date. Pediatrics patients are separated into sub-specialties as for adult services.

C.2.1.2 Condensed Billing

Prior to this study, the condensed billing information was deleted after each transaction and was not saved on a computer-accessible medium. A special request was submitted to save condensed billing information for another project from October 1975. Thus, at the time of this study condensed billing information was available for a period of six months. The hospital service in the condensed billing tape does not categorize the Pediatrics service into sub-specialties. The admission rate and discharge date were numbered from January 1, 1900, up. For example, admission date 0

corresponds to January 1, 1900, 1 to January 2, 1900, and 365 to December 31, 1900. All fields with numbers in the condensed billing tape were either in binary or hexadecimal codes. The condensed billing tape was decoded and desired information was selected.

The two tapes of patient abstract and condensed billing information were matched by medical record number and length of stay since each patient could be admitted several times. The steps involved in the extraction of desired data from these two tapes are presented in Figure C.1.

The length of stay data obtained were used in the statistical analysis study. The day of the week of admission was obtained by using a modulo arithmetic and based on January 1, 1900, being a Monday. Information on diagnoses was used to classify patients into single and multiple diagnoses. The length of stay distributions by service and by day of the week distributions were also extracted from the length of stay data.

C.2.2 Unscheduled Admission Data

The unscheduled admission data were collected by going through the admission summary sheets day by day. The number of unscheduled admissions for the Pediatrics unit was recorded by each service for a period of a year from May 1975 to April 1976. The mean values of unscheduled admissions by day of the week for all services were calculated.

C.2.3 Daily Data for the Census Prediction Model

Two kinds of information were needed daily for the census prediction process: The census count and the scheduled reservations for the next 15 days. The census count of the number of patients in each service by their lengths of stay was obtained from the floor midnight census report. The patients with length of stay more than 25 days were considered as 25-day length of

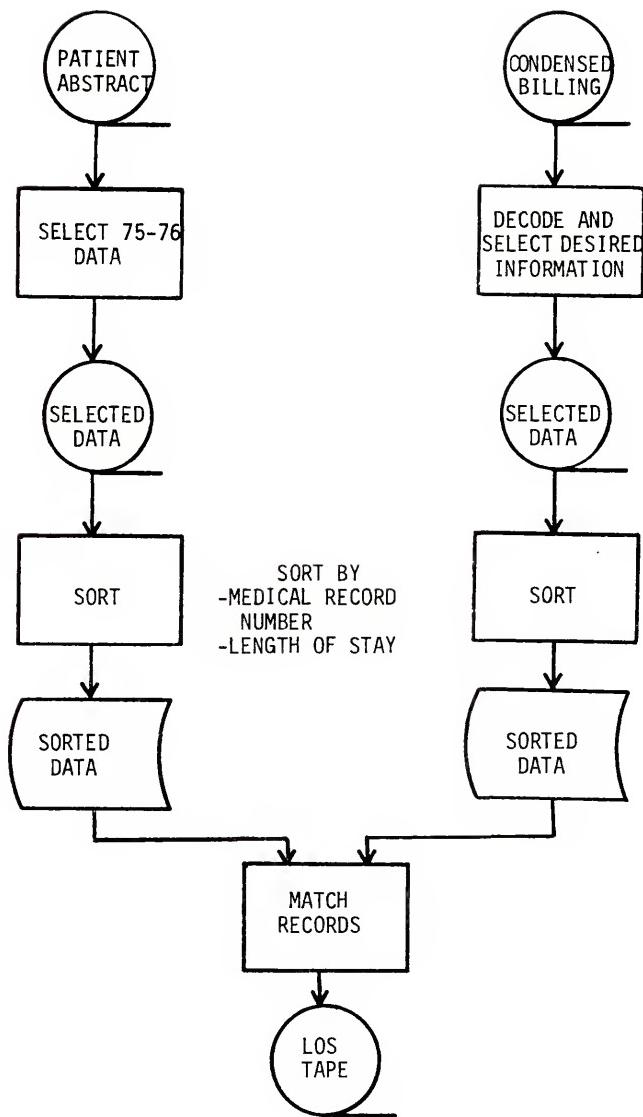


Figure C.1 Job steps in collecting length of stay data for Shands Teaching Hospital.

stay patients. The numbers of scheduled admissions for each day of the next 15 days by each service were provided by the ward secretary daily.

C.3 Gainesville Veterans Administration Hospital

C.3.1 Length of Stay Data

The length of stay data of the Gainesville Veterans Administration Hospital were extracted from the patient treatment file. A tape of the patient treatment file for fiscal year 1975 was obtained from the Data Processing Center of the Veterans Administration at Austin, Texas. The patient treatment file contains one admission transaction record, one or more diagnostic and surgical transaction records, and one disposition transaction record for each admission. The tape was under a variable record length format, each kind of record was identified by a transaction code. From the admission transaction record, the demographic information of patients such as age, sex, and date of admission was obtained. The diagnostic transaction recorded the ICDA diagnostic codes; each transaction could contain up to five diagnostic codes. The surgical transaction recorded the data of surgery and surgical specialty, and ICDA operative codes; each transaction could contain up to five operative codes. The disposition transaction consisted of information such as date of disposition, disposition status, and length of stay. Thus all necessary information for the length of stay could be obtained from the tape. Since there were extra information on the file, data from the tape were selected and reformatte to fixed length records for easier access.

C.3.2 Unscheduled Admission Data

There were no records on the unscheduled admissions at the Gainesville Veterans Administration Hospital. The data were collected by comparing

the daily summary of Gains and Loss in the hospital and the scheduled admission log sheets. The data were available from March to August 1976. Since the number of "no shows" for the hospital was negligible, this method of collecting of unscheduled data yielded reasonably accurate file of information.

C.3.3 Daily Data for the Census Prediction Model

Previous to this study, the Gainesville Veterans Administration Hospital did not have any census report. Information on patients currently in the hospital was kept on a card file. The file was usually not up to date and there was no information on the patient's service. Efforts were made to generate a midnight census report by ward. A census master file was built; admissions and discharges were collected daily to update the census file. Since the patient's service was not included with the patient's information, the admitting office had to provide the service for each admission. Every day, the floor census reports were generated and checked by the ward secretary for correct service and information. A flow chart representing the steps involved in obtaining the census count is in Figure C.2.

The scheduled admissions were logged in the admission sheet as the patient or physicians called in to report the scheduled date. Many scheduled admissions failed to report to the admitting office in advance of the admission day and were considered as unscheduled admissions in this study.

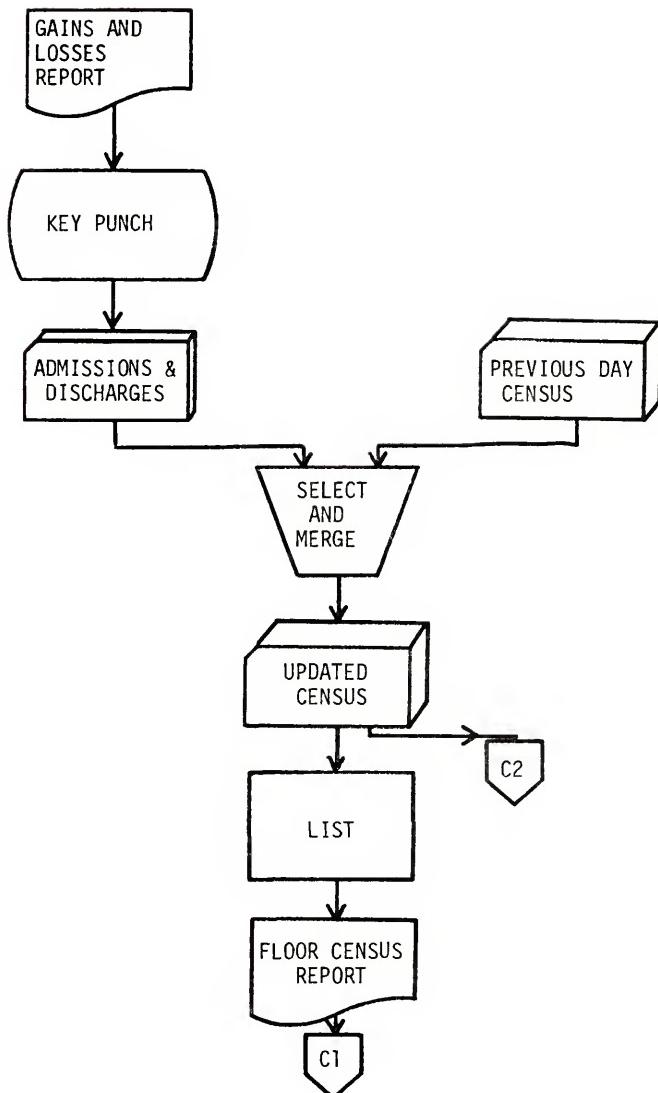


Figure C.2 Steps involved in creating census count for Gainesville Veterans Administration Hospital.

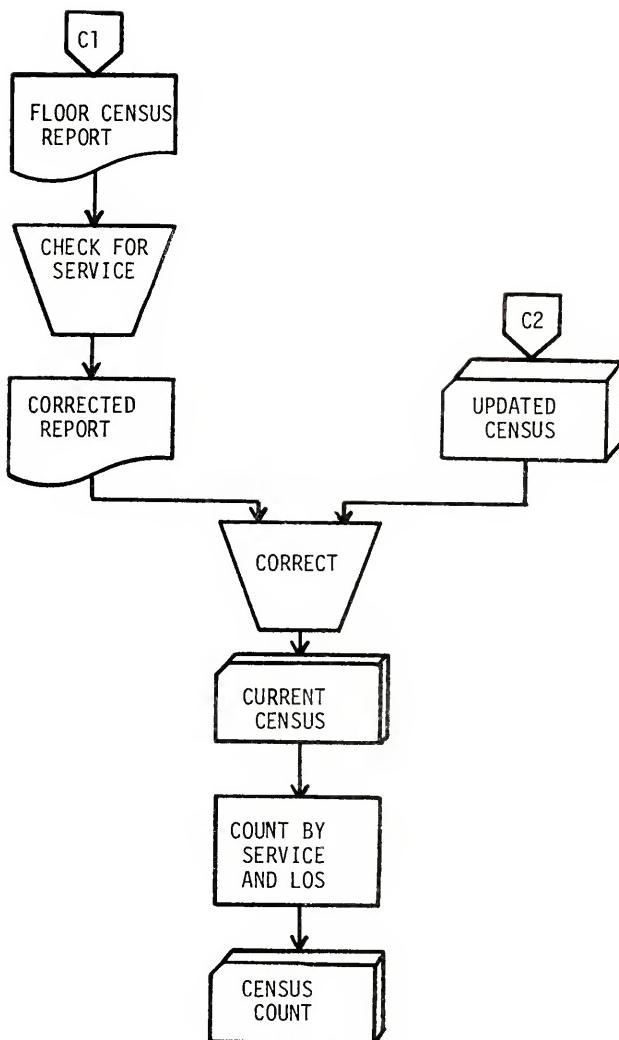


Figure C.2 (continued)

C.4 North Florida Regional Hospital

C.4.1 Length of Stay Data

The length of stay data were obtained from PAS case abstract file of the year 1975. The patient case abstract provided information on length of stay, admission date, discharge data, weekday of admission, hospital service, attending physician, sex, race, age on admission and type of admission--emergency, urgent, transfer from other hospitals, or through emergency room--along with other information not essential to the study. Data necessary for the length of stay were selected from the file together with the type of admission.

C.4.2 Unscheduled Admission Data

As described previously, the patient case abstract file contained information on the type of admissions. Therefore, the number of emergency, urgent, and direct admissions for each day of the year 1975 was obtained directly from the file. The mean values of the unscheduled admissions by day of the week and by service were calculated thereby.

C.4.3 Daily Data for Census Prediction Process

The Alpha census report listing for North Florida Regional Hospital was printed out daily but did not reflect the actual midnight census due to the delay in entering daily discharges and admissions. Efforts were made to generate the actual midnight census daily. A census master file was created and updated daily by the admissions and discharges. The admission and discharge data were entered to the computer through a GE terminal at the Hospital. The numbers of scheduled admissions by service for five days in advance were also entered to the terminal. Every day, the census file

was updated. The total census number was checked with the Admitting Office. The steps involved in producing census counts by service for the hospital is presented in Figure C.3.

C.5 Alachua General Hospital

C.5.1 Length of Stay Data

The length of stay data for Alachua General Hospital were obtained from the condensed billing file from September 1975 to October 1976. Information consisted of hospital service, length of stay, admission date, discharge date, age, sex, and race. The data system of Alachua General Hospital was similar to that of Shands Teaching Hospital since they both used the SHAS system.

C.5.2 Unscheduled Admission Data

The daily admission summary sheets were used to count the number of emergency, urgent and direct admissions for the period from September 1975 to October 1976. The mean values of unscheduled admissions were then obtained from these data.

C.5.3 Daily Data for Census Prediction Process

Alachua General Hospital had an on-line data processing system; therefore the midnight census report was generated daily together with the report of admissions, discharges and transfers. The census counts for census prediction process were obtained from the Alpha census report listing. The scheduled admissions for any day in the future were kept in a log book. The services were not recorded with the admission, therefore the service had to be derived from the admitting doctor.

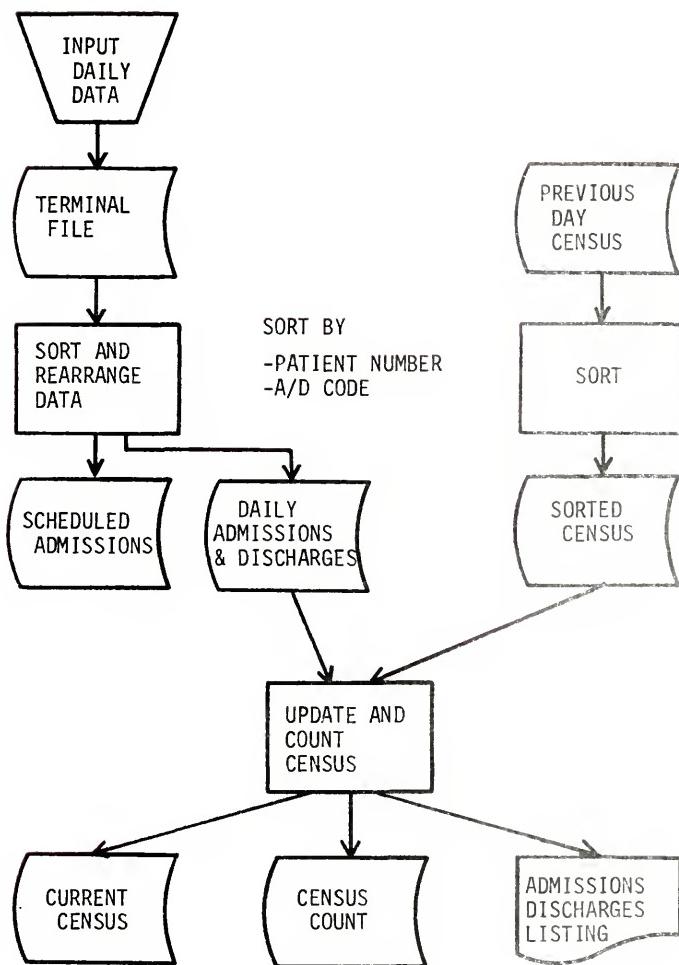


Figure C.3 Job steps for generating census count for North Florida Regional Hospital.

APPENDIX D

THE EXPECTED VALUE AND VARIANCE OF A RANDOM SUM Z_j OF INDEPENDENT BERNOULLI TRIALS

Let Z_j be the number of patients arrived on any particular day who will stay in the service exactly j days.

$$Z_j = x_1 + x_2 + \dots + x_A$$

where A = number of arrivals, random variable,

and

$$x_i = \begin{cases} 1 & \text{for arrivals with } P[L=j] , \\ 0 & \text{otherwise .} \end{cases}$$

$P[L=j]$ = probability that the length of stay is exactly j days.

The expected number of arriving patients who will stay exactly j days, given that the number of arrivals is 'a', is developed as follows

$$\begin{aligned} E[Z_j | A=a] &= \sum_{z=0}^a z P[Z_j=z | A=a] \\ &= \sum_{z=0}^a z \binom{a}{z} (P[L=j])^z (1-P[L=j])^{a-z} \end{aligned} \quad (D.1)$$

The right hand side of Equation (D.1) is just the expected value of a binomial distribution. Therefore

$$E[Z_j | A=a] = a P[L=j]$$

The expected value of the random sum Z_j is

$$\begin{aligned} E[Z_j] &= \sum_{a=0}^{\infty} E[Z_j | A=a] P[A=a] \\ &= \sum_{a=0}^{\infty} a P[L=j] P[A=a] \\ &= E[A] P[L=j] \end{aligned}$$

The variance of the random sum can be derived similarly.

$$\begin{aligned} E[Z_j^2 | A=a] &= \sum_{z=0}^a z^2 P[Z_j=z | A=a] \\ &= \sum_{z=0}^a z^2 \binom{a}{z} (P[L=j])^z (1-P[L=j])^{a-z} \\ &= a P[L=j] (1-P[L=j]) + a^2 (P[L=j])^2 \end{aligned}$$

The expected value of Z_j^2 , $E[Z_j^2]$, is

$$\begin{aligned} E[Z_j^2] &= \sum_{a=0}^{\infty} E[Z_j^2 | A=a] P[A=a] \\ &= \sum_{a=0}^{\infty} \{a P[L=j] (1-P[L=j]) P[A=a] + a^2 (P[L=j])^2 P[A=a]\} \\ &= E[A] P[L=j] (1-P[L=j]) + E[A^2] P^2[L=j] \end{aligned}$$

Thus

$$\begin{aligned} V[Z_j] &= E[Z_j^2] - (E[Z_j])^2 \\ &= E[A] P[L=j] (1-P[L=j]) + V[A] P^2[L=j] . \end{aligned}$$

APPENDIX E

PROGRAM LISTING FOR THE BOUNDS ON STATE PROBABILITIES
OF THE $M/E_k/s_j/s_{N+1}^*$ QUEUEING SYSTEM

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APPENDIX F

PROGRAM LISTING FOR THE
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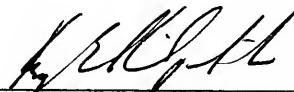
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BIOGRAPHICAL SKETCH

Khanh-Luu Thi Nguyen was born December 3, 1947, in North Vietnam. In June, 1965, she was graduated with High Honors from Trung Vuong High School in Saigon, South Vietnam. In January, 1966, she began undergraduate work at the University of Colorado in Boulder, Colorado, under a scholarship from the Department of Education of South Vietnam. She received the degree of Bachelor of Arts in Physics and Mathematics with cum laude in Physics in June, 1969. In September, 1969, she began graduate work in Physics under graduate fellowships at Harvard University. She received the degree of Master of Arts in Physics in June, 1970. Returning to Vietnam, she joined IBM-World Trade Corporation in Saigon, South Vietnam, as a Systems Engineer. In September, 1973, she came back to the U.S.A., and started graduate work in Systems Engineering at the University of Florida. In August, 1976, she received the degree of Master of Science in Industrial and Systems Engineering. From October, 1975, until the present, her graduate training has been supported by a Health Services Research and Development traineeship from the Veterans Administration.

Khanh-Luu is married to Khai Quoc Nguyen. She is a member of Phi Beta Kappa, Sigma Pi Sigma, Alpha Pi Mu, and Phi Kappa Phi honorary societies.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



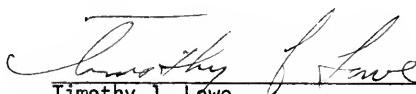
Kerry E. Kilpatrick, Chairman
Associate Professor and Director,
Health Systems Research Division

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Thom J. Hodgson, Co-Chairman
Associate Professor of Industrial and
Systems Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



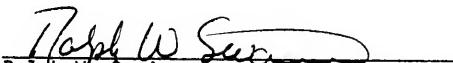
Timothy J. Lowe
Assistant Professor of Industrial and
Systems Engineering

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James T. McClave
Assistant Professor of Statistics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

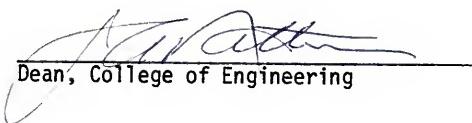


Ralph W. Swain

Associate Professor and Associate
Director, Health Systems Research Division

This dissertation was submitted to the Dean of the College of Engineering and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

June 1977



J.W. Arthur

Dean, College of Engineering

Dean, Graduate School

UNIVERSITY OF FLORIDA



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